Surface zonal flows induced by thermal convection trapped below a stably stratified layer in a rapidly rotating spherical shell

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Abstract. Penetration of finite-amplitude columnar convection into an outer stably stratified layer in a rapidly rotating spherical shell is examined numerically. It is shown that penetration of columnar convection is not always required for generation of surface zonal flows. When the strength of the stratification of the outer stable layer is increased, small-scale columnar convection cells are trapped below the layer, but induced mean zonal flows still penetrate to the surface. Our results suggest that the surface zonal flows of the giant planets may be a consequence of penetration of deep zonal flows generated by small-scale columnar convection trapped below a near-surface stably stratified layer.

1. Introduction

In order to investigate possible generation mechanisms of the alternating zonal flows and equatorial superrotation observed at the surfaces of the large gas planets according to the assertion by *Busse* [1976], direct calculations of three-dimensional columnar convection in rapidly rotating spherical shells have been performed [*e.g. Zhang and Busse*, 1988; *Ardes et al.*, 1997; *Tilgner and Busse*, 1997; *Sun and Schubert*, 1995; *Christensen*, 2001; *Aurnou and Olson* 2001]. The uniform mean zonal-flow distribution observed at depth beneath the cloud level by the Galileo probe supports the explanation of mean zonal-flow generation by deep convection [*Atkinson et al.*, 1996]. However, the Galileo probe also showed that the layer between 5 bar and 16 bar is stably stratified [*Seiff et al.*, 1996]. Stable stratification is also suggested in a thicker deeper layer by a theoretical calculation of the internal structure of the large gas planets [*Guillot et al.* 1994]. If such a stable layer exists globally then deep trapped convection might not explain the generation of surface zonal flows. The extent of penetration of deep fluid motion is thus an important point to settle in order to find whether this generation mechanism can operate or not.

Penetration of columnar convection may also play an important role in the dynamics of the Earth's fluid core. The possible existence of a stably stratified layer in the outer core has been discussed by many authors [*e.g. Whaler*, 1980; *Fearn and Loper*, 1981; *Braginsky*, 1984; *Gubbins et al.*, 1982; *Labrosse et al.*, 1997; *Lister and Buffett*, 1998]. Oscillations and fluid motion in such a stable layer has been suggested to be the origin of the Earth's secular variation [*e.g. Braginsky*, 1984; *Braginsky*, 1993; *Yokoyama and Yukutake*, 1993]. However, flow in this outer layer is not necessarily independent of that in the underlying fluid, since convective motion may penetrate the stable layer from below. The extent of penetration of columnar convection is one of the key points to consider in the dynamics of the fluid motion in any stable layer, which might be related not only to the secular variation but also to the geodynamo.

Systematic studies of the penetration of columnar convection into an outer stable layer in a rapidly rotating spherical shell have been performed only for critical convection (i.e. at linear onset). There have been no systematic studies of finite-amplitude convection. *Yano* [1985] extends *Busse* [1983] to a two-layer model and investigates critical convection with an asymptotic expansion method. *Zhang and Schubert* [1996, 1997] examine critical convection numerically and illustrate complete penetration of columnar convection to the outer stable layer. *Takehiro and Lister* [2001] derive an analytic expression for the penetration distance of columnar motion from the dispersion relation of inertia-gravity waves and confirm it with numerical calculations of critical convection. *Zhang and Schubert* [2000] find different forms of penetration of critical convection at smaller Prandtl numbers. Here, we perform numerical experiments for finite-amplitude convection in a rapidly rotating spherical shell with an outer stably stratified layer. We investigate the distribution of mean zonal flow induced by columnar convection for various strengths of stratification.

2. Model

We consider the case of a self-gravitating body with a constant angular velocity. The basic state and governing equations are the same as those of *Takehiro and Lister* [2001] except that the nonlinear terms are now included in the equations of motion and heat transfer. The equations are normalized by the thickness d of the shell and the thermal diffusion time d^2/κ, where κ is the thermal diffusivity. The nondimensional parameters in the system are the Rayleigh number $R = \alpha g_o \varepsilon d^5/3K\kappa\nu$, Taylor number $T = 4\Omega^2 d^4/\nu^2$, and Prandtl number $P = \nu/\kappa$, where ν is kinematic viscosity, α is the thermal expansivity, g_o is gravity at the outer radius, Ω is the angular velocity of the shell, K is thermal

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conductivity and ε is the homogeneous heat source, which produces unstable stratification in the inner part of the shell. The nondimensionalized temperature gradient of the basic state is

$$\frac{d\Theta}{dr} = -\frac{1}{2}(r + \Gamma_0)\left[1 - \tanh\left(\frac{r - r_b}{a}\right)\right] + \Gamma_0. \quad (1)$$

The tanh profile achieves a smooth match from an unstable gradient $-r/2$ to a stable gradient Γ_0 over a narrow transition zone of thickness a centered at radius r_b. The imposed boundary conditions are free-slip and fixed temperature.

In the numerical calculations, toroidal and poloidal potentials were introduced to describe the incompressible velocity field and remove the pressure. The variables were expanded in spherical harmonics in the horizontal directions, while radial derivatives were evaluated by Chebyshev collocation with 33 radial grid points. 128 and 64 grid points were placed in longitude and latitude, respectively, and the total horizontal wavenumber was truncated at 42. The nonlinear terms were evaluated in grid space and transformed back to spectral space. Time integration was performed by a second-order Runge-Kutta scheme except that the diffusion terms were modeled by a Crank-Nicolson scheme.

All calculations were performed for $P = 1$ and $T = 10^8$. We fixed the radius ratio at $\eta = 0.4$, which gives inner and outer radii at $r_i = 0.67$ and $r_o = 1.67$; r_b and a were fixed at 1.2 and 0.05, respectively. The strength of stratification Γ_0 was varied from 1 to 500. The Rayleigh number was roughly four times the critical value for each case. A temperature disturbance with an amplitude of 0.01 was placed at a point in the middle of the shell as a initial condition. Time integrations were performed until the convective motion was fully developed and the kinetic energy saturated.

3. Results

Fig. 1 shows a snapshot of fully developed convection for $R = 2 \times 10^6$ and $\Gamma_0 = 10$. Pairs of equatorially antisymmetric vortices corresponding to columnar convection cells can be seen in Fig. 1(a). From the equatorial cross section, we find that convection cells with longitudinal wavenumber 7 emerge in the inner unstable layer. However, an azimuthal velocity is induced through the whole shell (Fig. 1c).

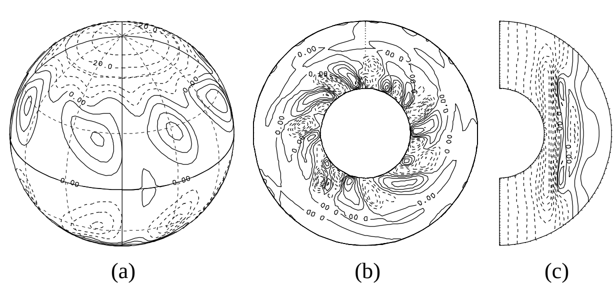

Figure 1. A snapshot of fully developed thermal convection for the case $P = 1, T = 10^8, R = 2 \times 10^6$ and $\Gamma_0 = 10$. Solid and broken lines indicate positive and negative values, respectively. (a) Toroidal potential field at the top of the unstable layer ($r = 1.1$). (b) The radial component of velocity in the equatorial plane. (c) The azimuthal component of velocity in a meridional cross section.

Fig. 2 shows distributions of the azimuthal component of the velocity field for various strengths of stable stratification. The extent of penetration of small-scale columnar convection cells can be seen from the distributions of the disturbance fields. For $\Gamma_0 = 1$, the convection columns penetrate the stable layer completely. However, the extent of penetration decreases by $\Gamma_0 = 10$. When Γ_0 is further increased to 50 or larger, convection columns are largely confined to the lower unstable layer. This is consistent with results for critical convection in *Takehiro and Lister* [2001]. Their theoretical estimate of penetration thickness is thus also expected to apply to finite-amplitude columnar-type convection.

For $\Gamma_0 = 1$, the mean zonal flow is axially uniform through both layers and penetrates to the surface without noticeable decrease in amplitude. This zonal flow is produced by the Reynolds stress associated with the tilting structure of the columns [*Busse*, 1983], as seen in Fig. 1(a,b). Since columnar convection also penetrates the layer in this case, the zonal flow in the stable layer is generated directly in situ by the local columnar convection. As Γ_0 is increased from 10 to 100, the strength of mean zonal flows in the stable layer slightly decreases, but they still penetrate the stable layer to the surface. Since columnar convection becomes trapped below the stable layer, these surface zonal flows are the result not of in situ generation by the columnar convection, but of remote generation from the deep region. In the inner unstable regions, the Reynolds stress of small-scale columnar convection produces deep zonal flows. These deep zonal flows penetrate the stratified layer to the surface but the columnar convection itself does not. Note that the distribution of mean zonal flow in the inner unstable layer is uniform in the direction of the rotation axis and similar to the whole distribution of zonal flow for $\Gamma_0 = 1$.

One explanation of the emergence of remotely generated surface zonal flows might be penetration by inertia-gravity wave propagation. *Takehiro and Lister* [2001] argue that stratification acts as a low-pass filter to forcing from below, so that large-scale components can penetrate the stable layer while the small-scale components are trapped below the layer. This argument seems to be valid for $\Gamma \leq 50$ in our results. However, the lack of dependence of the penetration distance on Γ_0 when the strength of stratification is further increased cannot be explained by this mechanism. Mean zonal flows still penetrate the stable layer for $\Gamma_0 \geq 100$ and their distribution barely changes with Γ_0. In order to show this more clearly, Fig. 3 compares the penetration thicknesses of mean zonal flow estimated by the theoretical expression (14) of *Takehiro and Lister* [2001] with that derived from the numerical calculations. A notable feature is that the numerical penetration thickness is almost independent of the stratification. We suggest that penetration of mean zonal flows in these cases is not governed by propagation of inertia-gravity waves, but by viscous diffusion, which is neglected in the analysis of *Takehiro and Lister* [2001]. This is supported by the observations that the extent of penetration is almost same as the horizontal scale of the mean zonal flow and that the time scale of mean zonal flow development is similar to the viscous diffusion time of the stratified layer ($t \sim 0.3$).

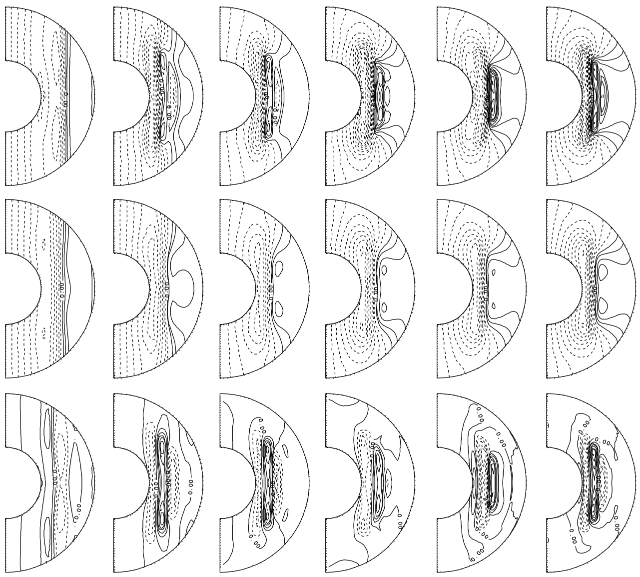

Figure 2. Distributions of the azimuthal component of the velocity fields at $t = 0.5$ for various strengths of stable stratification. Upper, middle and lower rows are the total, zonally averaged and disturbance fields, respectively. (a) $\Gamma_0 = 1$, $R = 10^6$, (b) $\Gamma_0 = 10$, $R = 2 \times 10^6$, (c) $\Gamma_0 = 50$, $R = 2 \times 10^6$, (d) $\Gamma_0 = 100$, $R = 3 \times 10^6$, (e) $\Gamma_0 = 200$, $R = 3 \times 10^6$, (f) $\Gamma_0 = 500$, $R = 4 \times 10^6$. The outer half region of the shell is stably stratified. The contour interval is 10. Solid and broken lines indicate positive and negative values, respectively.

However, penetration in the early stages of convection is still governed by the mechanism of inertia-gravity waves because it takes longer for the establishment of viscous penetration than the time scale for development of convective motion and penetration by inertia-gravity waves. The initial exponential increase of total kinetic energy is finished within $t = 0.1$ in all cases, but in the cases of strong stratification the system has not by then reached a statistically steady state. In the strongly stratified cases the kinetic energy increases gradually until $t = 0.5$ due to spin up in the stable layer by viscous coupling. On the other hand, the time scale for penetration by inertia-gravity waves is very short; it is similar to the propagation time of the waves across the stable layer, which is of order the rotation period of the system, $1/\sqrt{T} \sim 10^{-4}$. Thus, penetration by inertia-gravity waves is expected to be finished by the time that convective motion in the unstable layer is developed. Fig. 3 shows that the thickness of mean zonal flows in the early stages is well explained by the analytic prediction for inertia-gravity wave penetration.

Viscous diffusion does not play an important role in the penetration of small-scale columnar convection, due to the propagation of the convection column. Through the tilting of the spherical outer boundary, the columnar convection cells propagate eastward by the topographic Rossby wave mechanism and the effect of diffusion is averaged out. The extent of viscous penetration of such propagating vortical motions is estimated by the Stokes layer thickness $\delta_\nu^* = \sqrt{\nu/\omega^*}$, where ω^* is the propagation frequency of the vortical motions, or in non-dimensional form $\delta_\nu = 1/\sqrt{\omega}$. From the calculation of critical convection, the frequency of Rossby waves in the calculations is $\omega \sim 200$. This gives $\delta_\nu \sim 0.07$, which is small compared to the penetration distance by the inertia-gravity wave mechanism. On the other hand, viscous penetration of the mean zonal flow is effectively large because of its non-oscillatory character ($\omega = 0$).

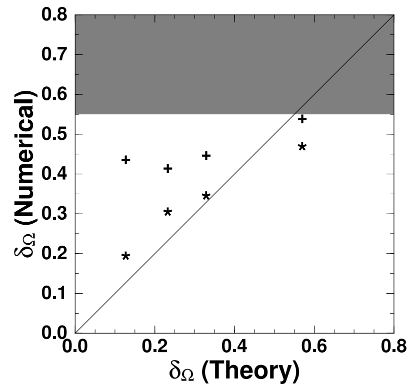

Figure 3. Comparison of penetration thicknesses of mean zonal flows. The horizontal and vertical axes show the thickness $\delta_\Omega = 2\Omega/N|\boldsymbol{k}|$ derived by *Takehiro and Lister* [2001, Eq.14] and that derived from the numerical calculations, respectively. Here N is the Brunt-Väisälä frequency in the stratified region and $|\boldsymbol{k}|$ is the modulus of the horizontal wavenumber. Stars denote the early stages where the initial exponential development is finished ($t = 0.1$) while crosses denote the fully developed states ($t = 0.5$). The penetration thickness cannot be larger than the thickness of the stable layer (hatched region).

4. Discussion

From the present results, the extent of penetration and the penetration mechanisms of columnar fluid motions in a rapidly rotating spherical shell are summarized as follows. Two penetration mechanisms are available, namely diffusion and inertia-gravity waves, which have different time scales and different dependence on horizontal scales. As analyzed by *Takehiro and Lister* [2001], the penetration thickness by inertia-gravity waves is proportional to the angular velocity of the system and the horizontal scale of the vortices, and inversely proportional to the Brunt-Väisälä frequency in the stratified layer. This penetration mechanism operates over the short time scale of the rotation period. On the other hand, penetration by viscous diffusion operates on a much longer time scale and over a potentially greater distance. It operates effectively on the non-oscillatory motions, and allows zonal flows to penetrate the stratified layer to a distance comparable to their horizontal scale. However, diffusion does not allow small-scale columnar convective motions to penetrate the stratified layer due to their oscillatory character.

By using the two scalings of penetration thickness, we can examine whether deep zonal flows can penetrate a surface stable layer or not. For example, a horizontal scale larger than O(1000) km is required for complete penetration by inertia-gravity waves through the stable layer observed by the Galileo probe [*Takehiro and Lister*, 2001]. If the viscous diffusion time of the layer is sufficiently short for penetration by diffusion to operate, the necessary horizontal scale of zonal flows for penetration is only O(100) km, which is similar to the thickness of the stable layer. *Both mechanisms are consistent with* the suggestion that the observed surface zonal flows of Jupiter originate from the deep region and penetrate the stable layer, because the width of the alternate banded structure of Jupiter is $O(10^4)$ km.

The most interesting feature of the present results is the existence of finite-amplitude solutions where surface zonal flows are generated remotely by columnar convection trapped below the stratified layer. Thus penetration of columnar convection itself is not required for generation of surface zonal flows. This result suggests that surface zonal flows of the giant planets may be a consequence of penetration of deep zonal flows generated by unobservable small scale columnar convection trapped below the stably stratified layer. Note that this mechanism contrasts with the suggestion of the linear solutions of *Zhang and Schubert* [2000] at low Prandtl numbers, whereby thermal convection in the unstable inner layer remotely drives small scale convective motion in the stratified layer, which may lead to in situ generation of large scale surface zonal flows.

We note that our results are based on slightly supercritical calculations and further work should examine how far the scalings carry over to highly nonlinear large-amplitude convection. We also note that the effect of compressibility should be considered for detailed application to the Jovian atmosphere since the analysis here is based on the Boussinesq system. For example, further work could use the anelastic approximation. However, we expect that the leading-order behaviour is captured by use of potential temperature and mass flux instead of temperature and velocity, respectively. Thus, compressibility would simply enhance penetration of the fluid motion from the deep region.

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