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A simple dynamical model for gravity drainage of brine from growing sea ice

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[1] Gravity drainage of brine through liquid brine channels is the dominant mechanism for the desalination of growing sea ice. We describe and determine mathematically the essential physics of this process, elucidating the connection between downward flow in brine channels and a convective upward flow in the rest of the porous ice, which we show has a vertically linear structure and strength proportional to a Rayleigh number. Our simple dynamical model of this process is used to interpret the exponential propagation of dye fronts in laboratory experiments. We propose that using our new, derived parameterization for gravity drainage in sea ice in terms of two unknown parameters could lead to computationally feasible improvements to thermodynamic sea-ice models.
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1. Introduction

[2] The polar seasonal cycle sees an enormous volume of sea ice frozen each year. As leads open up and refreeze, and as pack ice thickens over the winter, salt water is initially held within a matrix of porous sea ice. The interstitial brine becomes increasingly concentrated in salt, which is segregated from the solid phase as the ice continues to freeze. This increasingly dense brine can then drain from the ice under the action of gravity, both reducing the salinity of the ice and causing plumes of dense brine to sink into the polar oceans, leading to bottom-water formation and vertical mixing. It is important to understand this process of gravity drainage, along with other mechanisms such as flooding and flushing that affect the salinity of sea ice, in order to determine the thermal and mechanical properties of sea ice [reviewed in *Weeks*, 2010, ch. 10], and also to model the polar climate system [*Holland et al*., 2006; *Vancoppenolle et al*., 2009].

[3] However, while the significance of gravity drainage to brine fluxes from sea ice during its winter growth has long been acknowledged [*Untersteiner*, 1968; *Niedrauer and Martin*, 1979; *Notz and Worster*, 2009], it has proved difficult to incorporate this process directly in the sea-ice component of Global Climate Models in a sufficiently simple fashion [*Hunke et al*., 2011]. Indeed, even the most resolved, established sea-ice models (such as CICE: the Los Alamos sea ice model) prescribe the bulk salinity of the ice, use it to calculate its thermal properties, and then calculate the thermodynamic growth of the ice [*Maykut and Untersteiner*, 1971; *Bitz and Lipscomb*, 1999]. Although this approach is a reasonable starting point, a dynamically informed model to predict the bulk salinity would constitute a major advance and would increase confidence in the predictions of Global Climate Models in significantly changed climatic conditions in which the proportion of first-year ice might be much higher and the previously prescribed salinity profiles, which are more appropriate to multi-year ice, might consequently be less appropriate.

[4] Some recent theoretical studies approached this challenge by treating sea ice as a two-phase reactive porous medium and solving partial differential equations for heat, salt, and mass conservation, using Darcy's law for the interstitial fluid flow. *Oertling and Watts* [2004] and *Wells et al*. [2011] constitute important, contrasting studies in this vein.

[5] In this paper, we describe a simple theoretical model that dynamically captures gravity drainage through narrow liquid brine channels within the ice. In particular, we calculate the strength of the interstitial upwelling away from the main channels required to replace the interstitial liquid that flows into the brine channels and thence into the ocean. We then use our results to interpret the classic experiments of *Eide and Martin* [1975] concerning dye-front propagation in laboratory-grown sea ice, which provides a consistency check for our model.

2. Physical Description

[6] Fluid flow in sea ice associated with gravity drainage is not restricted to liquid brine channels; rather, that flow is part of a convective circulation that occurs throughout the porous ice, since the brine-rich liquid that leaves the ice through brine channels must necessarily be replaced by liquid flowing into the ice from the ocean. Therefore, there is a net upwelling in the bulk of the ice, a phenomenon that has been observed by *Eide and Martin* [1975], who describe this as entrainment, but whose significance is arguably under appreciated.

[7] In growing ice, convection is sustained by the following physical mechanism. The ice near a brine channel (both the solid matrix and interstitial brine) is cooled by conduction from the cold liquid flowing down the channel; relatively cold interstitial brine is also relatively concentrated, since the freezing temperature of salt water decreases with salinity, and to a very good approximation the interstitial brine is at local thermodynamic equilibrium [*Feltham et al*., 2006]; this establishes the horizontal density gradient of the interstitial

All Supporting Information may be found in the online version of this article.

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brine that sustains convection, and we determine this flow mathematically in section 3.

[8] While postmortems of sea ice often reveal a brine drainage network that persists through much of its depth [e.g., *Lake and Lewis*, 1970], there is evidence (for instance, *Eide and Martin* [1975], discussed in section 4) that after an initial transient period when convection leading to brine transport can occur within the whole depth of ice, convection is confined to a relatively thin layer at the bottom of the ice, as indicated in Figure 1. It has been suggested that above this layer the ice has a porosity below some critical porosity (say 5%) at which the permeability of the ice drops essentially to zero [*Golden et al.*, 2007]. The observation of confinement of the flow is, alternatively, also consistent with a suggestion that the depth of the convecting layer is set by a critical Rayleigh number that depends on both the permeability and the local temperature gradient. This interpretation was made following the field experiments of *Notz and Worster* [2008], developing the previous observation that delayed onset of gravity drainage is controlled by a critical Rayleigh number [*Worster*, 1997; *Wettlaufer et al.*, 1997].

[9] The flow within a brine channel itself is only part of the overall mechanism of gravity drainage in sea ice. While the channel flow has received much attention (for example, *Lake and Lewis* [1970] use the theoretical study of convection in a semi-closed pipe by *Lighthill* [1953] to interpret flow in a brine channel, which is not appropriate since the surrounding ice is a porous medium), continuity requires that any description of gravity drainage must include the sustaining convective flow in the bulk of the ice.

3. Mathematical Model

[10] Here we present a simple framework in the context of idealized governing equations to determine the structure of this sustaining convective flow.

3.1. Governing Equations

[11] In the sense that sea ice is a two-phase reactive porous medium, it constitutes a mushy layer [*Feltham et al.*, 2006]. The mushy-layer equations [*Worster*, 1992, 1997] adopt the approach of continuum mechanics in averaging equations of heat, salt, and mass conservation over the two phases, using Darcy's law for the interstitial fluid flow, described by the Darcy velocity $\boldsymbol{u}$ and pressure p. The temperature T and *interstitial* brine salinity C are coupled by local thermodynamic equilibrium, and we assume that the associated liquidus relationship is linear. This allows us to introduce a single dimensionless variable for both: $\theta=(T-T_0)/\Delta T=-(C-C_0)/\Delta C$, where $\Delta T=T_0-T_a$ and $\Delta C=C_a-C_0$ are the temperature and interstitial salinity differences across the whole depth of ice, respectively, as shown in Figure 1. Taking the idealizations discussed below, we use the steady ideal mushy-layer equations [*Worster*, 1997], which are given *non-dimensionally* by

$$\Omega\boldsymbol{u}\cdot\nabla\theta=\nabla^2\theta, \tag{1}$$

$$\nabla\cdot\boldsymbol{u}=0, \tag{2}$$

$$\boldsymbol{u}=Ra(-\nabla p+\theta\boldsymbol{k}), \tag{3}$$

where $\boldsymbol{k}$ is a unit vector in the vertical direction.

[12] The group $\Omega=1+\mathcal{L}/(-T_a c_p)$, where c_p is the specific heat capacity of the ice and $\mathcal{L}$ is the latent heat of solidification, is the dimensionless factor by which the effective heat capacity of sea ice is enhanced by phase change [*Huppert and Worster*, 2012]. In particular, Ωc_p is a simplified form of the dimensional effective heat capacity derived by *Feltham et al.* [2006].

[13] The important dimensionless Rayleigh number, which represents the ratio of available potential energy for convection to diffusive and dissipative effects, is defined by

$$Ra=g\beta\Delta CKH/\nu\kappa, \tag{4}$$

where g is acceleration due to gravity, κ is the thermal diffusivity, ν and ρ are the kinematic viscosity and density of water, respectively, $\beta=\rho^{-1}\partial\rho/\partial C$ is a solutal expansion coefficient, and K is a typical permeability. The permeability depends on the local porosity of the convecting layer; however, for practical applications of our model, K can be taken as a mean value over the convecting layer [cf. *Notz and Worster*, 2008]. We use a quasi-steady approximation in which the growth rate $\dot{h}$ is constant on the timescale of convective turnover in which case an appropriate vertical length scale is $H=\kappa/\dot{h}$.

[14] The idealizations made in (1)–(3) isolate the mechanism of gravity drainage, which is the dominant mechanism of desalination in growing sea ice [*Untersteiner*, 1968; *Notz and Worster*, 2009; *Weeks*, 2010]. Thus, we neglect the diffusion of salt, which accounts for brine pocket migration but is a very slow process [e.g., *Untersteiner*, 1968]. Furthermore, we neglect all differences in the properties of the phases, including in density, which accounts for brine expulsion [*Bennington*, 1963]. This process causes weak redistribution of salt within the ice but causes no salt flux from the ice [*Notz and Worster*, 2009]. Finally, we assume that $\dot{h}\ll w$, the vertical Darcy velocity, such that the dominant balance in the heat equation is between conduction and convective transport. The resulting equations (1)–(3) provide the simplest meaningful mathematical description of convection within sea ice.

[15] Given the assumptions underlying the ideal mushy-layer equations, the equation expressing conservation of salt

$$\frac{\partial S}{\partial t}=-w\frac{\partial C}{\partial z} \tag{5}$$

is decoupled from equations (1)–(3) describing buoyancy-driven flow [cf. *Worster*, 1997]. In this equation, $S=(1-\phi)C$

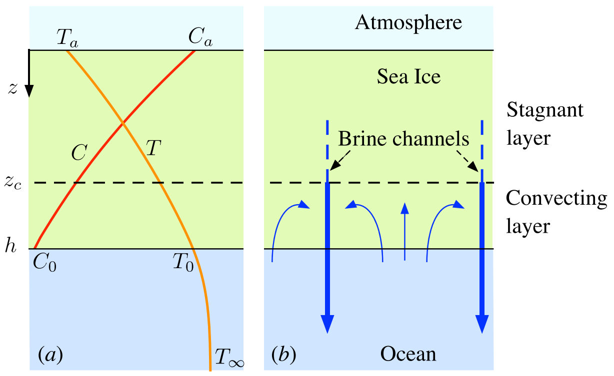

Figure 1. A schematic diagram of a one-dimensional model of sea ice: (a) typical temperature T and *interstitial* salinity C profiles; (b) sketch of convective flow within the ice (thin arrows) and down through brine channels into the ocean (thick arrows). Often this flow only occurs in a lower convecting layer between $z=z_c$ and $z=h$, as indicated. However, this is not a restriction imposed by our model.

is the bulk salinity, since we may neglect the salt content of solid ice which is very small, and we have also neglected molecular diffusion of salt as discussed above. Then, having determined the upwelling velocity w as we describe below, equation (5) can be used to determine the evolution of bulk salinity field and hence the solid fraction.

3.2. Channel-Active-Passive (CAP) Model

[16] The CAP model provides a simple characterization of convective solutions to (1)–(3). Full mathematical details are presented in *Rees Jones and Worster* [2013]; here we adapt our previous results from steady, directional solidification to the transient growth of sea ice. The CAP model can be applied in both two and three dimensions. For clarity, we sketch this approach applied to a periodic planar array of brine channels of separation $2L$. We introduce a stream function ψ such that $\boldsymbol{u}=(-\psi_z, \psi_x)$ satisfies the mass conservation equation (2). Throughout this paper, subscripts x and z denote partial derivatives. Then by taking the curl of (3),

$$\nabla^2\psi = -Ra\theta_x. \tag{6}$$

[17] Figure 2 shows how we divide up our periodic domain into the brine channel, the active region and the passive region to facilitate analytic progress. Firstly, in the *passive region*, which is defined by imposing that the temperature there is horizontally uniform ($\theta_x=0$), we take the temperature to be vertically linear, which is appropriate for a relatively thin convecting layer. Therefore,

$$\theta = \theta_0(z-h)/(z_c-h), \quad z_c \le z \le h, \tag{7}$$

where $\theta_0=-\Delta C_e/\Delta C$, in which $\Delta C_e=C_c-C_0$ is the interstitial brine salinity difference across the convecting layer. Horizontal uniformity means that (6) simplifies to Laplace's equation $\nabla^2\psi=0$. Vertically linear solutions for ψ correspond to horizontally uniform upwelling velocities, which are consistent with horizontally uniform solutions for θ. Therefore, the most general solution of this form that satisfies no flow through the periodic boundary at $x=L$ or at the top of the convecting region at $z=z_c$ is

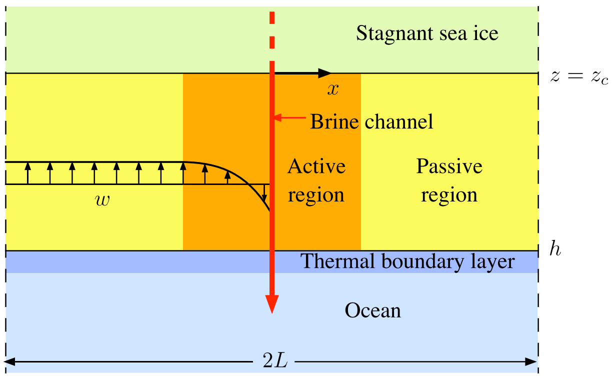

Figure 2. The Channel-Active-Passive (CAP) model. The vertical component of Darcy velocity w is uniform in the passive region but changes in the active region owing to the horizontal density gradient driving convection. Note that the shape of the isotherms is similar to the profile of vertical velocity: the temperature is horizontally uniform in the passive region and lower in the active region neat the brine channel.

$$\psi \propto (L-x)(z-z_c), \tag{8}$$

which corresponds to horizontally uniform upwelling.

[18] Secondly, the overall strength of the flow, or equivalently the proportionality factor in (8), is determined by matching this uniform upwelling in the passive region to an analytical solution of equations (1) and (6) in the *active region* near the chimney. This is defined as the region in which the temperature is not horizontally uniform ($\theta_x \neq 0$). The horizontal temperature gradient corresponds to a horizontal density gradient so there are active buoyancy forces in this region driving convection. Inside the brine channel itself, we follow *Chung and Worster* [2002] in determining the flow analytically using a lubrication (narrow-chimney) approximation.

[19] The salt flux from the ice into the ocean depends on channel spacing L. We find that there is both a minimum channel spacing [*Wells et al.*, 2010] and also a minimum width of the active region needed in order to sustain gravity drainage through channels. Therefore, horizontal density gradients in the surrounding ice sustain gravity drainage through channels. However, the mean channel spacing L remains undetermined. We follow the recent suggestion of *Wells et al.* [2010, 2011] that L takes the value that maximizes the salt flux. This avoids prescribing L, which is not an external parameter, and importantly allows us to determine the solution completely in terms of the single proportionality factor $W(\Omega)$ which can be computed cheaply. However, since the model includes a number of idealizations, for quantitative implementation we propose tuning this factor with experimental results.

[20] In conclusion, the dimensional vertical component of Darcy velocity w in the passive region, which gives a measure of the mean upwelling outside the brine channel, is

$$w = -\frac{\kappa}{h-z_c} Ra_e \frac{z-z_c}{h-z_c} W(\Omega), \tag{9}$$

where

$$Ra_e = [g\beta\Delta C_e K(h-z_c)]/\nu\kappa \tag{10}$$

is an effective Rayleigh number across the convecting layer. By integrating equation (5) for local salt conservation using equation (9) for the upwelling velocity w, we can determine the net salt flux from the mushy layer due to gravity drainage.

3.3. Comparison with Some Alternative Models

[21] These conclusions contrast with some recent suggestions about parameterizations of gravity drainage. They are fundamentally different to enhanced molecular diffusion or mixing-length diffusion [*Vancoppenolle et al.*, 2010; *Jeffery et al.*, 2011]; gravity drainage is an advective process. Consistent with this, our advective parameterization always transports salt (and any passive tracers) in the direction of the fluid flow and necessarily desalinates ice. By contrast, diffusive parameterizations imply down-gradient transport.

[22] Our model is closer to the prescription of a vertical velocity proportional to a Rayleigh number proposed by *Petrich et al.* [2011]; our mathematical modeling provides a solid justification for this kind of approach. However, whereas they impose a vertically uniform vertical velocity, we determine a linear structure (9). Our concept of the passive region matched to the active region around brine

channels shows theoretically that horizontally uniform vertical velocity corresponds to vertically linear vertical velocity. The concept of *Petrich et al*. [2011] requires much of the interstitial liquid to enter the brine channel at the top of the convecting region by conservation of mass. This seems unlikely both given the description of the nature of convection we argued for in section 2 and also given the observed tributary structure to brine drainage systems [e.g., *Lake and Lewis*, 1970]. Furthermore, the linear structure to the vertical component of the Darcy velocity given in (9) that we found is consistent with the experimental observations of *Eide and Martin* [1975] and *Chen* [1995], as we now demonstrate.

4. Analysis of Previous Experiments

[23] *Eide and Martin* [1975] investigate the interstitial flow by injecting dye into the liquid beneath growing ice in a laboratory, observing its horizontally uniform "entrainment" into the ice. For two cases, they measure the average height of the dye front as a function of time, which we reproduce in Figure 3, to which they fit exponential curves of the form $a[1-exp(-bt)]$.

[24] This exponential time dependence is explained by our simple model and fundamentally arises from the linear vertical structure we found for the vertical velocity. Neglecting diffusion of the dye, the dye front $z_d(t)$, which satisfies $z_d(0)=h$, is governed by

$$\frac{dz_d}{dt} = w(z_d, t) = -b(z_d - z_c), \tag{11}$$

using (9), where $b = Ra_e W(\Omega)\kappa/(h-z_c)^2$. This equation can be integrated immediately, assuming that the depth of the convecting layer $h-z_c$ evolves slowly compared to the dye-front position z_d. Together with the initial condition,

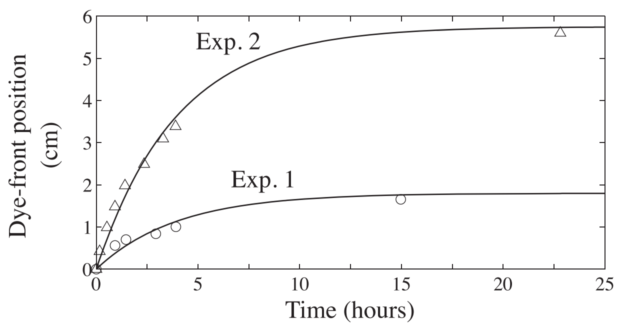

Figure 3. Reproduced from *Eide and Martin* [1975]. Height of dye front above interface height for two experiments. The exponential fits of the form $a[1-exp(-bt)]$ are part of the original figure. For both experiments, $b = 7\times10^{-5}\,\mathrm{s}^{-1}$, while $a = 1.8$ cm for experiment 1 and $a = 5.75$ cm for experiment 2 (when the ice was three times thicker). That b is approximately unchanged hints at the possibility that the permeability K has increased to compensate for the likely slackening of the brine salinity gradient $\Delta C_e/h-z_c$. Combined with the increase in $a = h-z_c$, equation (14) suggests that the effective Rayleigh number Ra_e increased. However, it is difficult to draw comparisons between only two experiments, especially as in the first experiment the ice was growing sufficiently rapidly that the quasi-steady approximation is unreliable.

integrating (11) shows that the height of the dye front above the injection point is

$$h - z_d = (h - z_c)[1 - \exp(-bt)]. \tag{12}$$

[25] This exponential time dependence contrasts, for example, with the piecewise linear time dependence that results from the model of *Petrich et al*. [2011]. Equation (12) provides a simple interpretation of a and b in *Eide and Martin* [1975]:

$$a = h - z_c, \tag{13}$$

the depth of the convecting layer, and, on rearrangement,

$$b = Ra_e \frac{\kappa}{(h-z_c)^2} W(\Omega) \equiv \frac{g\beta}{\nu}\frac{\Delta C_e}{h-z_c} K W(\Omega). \tag{14}$$

[26] Further confirmation of the linear vertical structure in (11) from the experiments of *Chen* [1995] on a different physical system is presented in the Auxiliary Material.

5. Conclusions

[27] We have derived a new parameterization for gravity drainage in sea ice theoretically in terms of two unknown parameters. Our mathematical modeling and experimental comparison indicate the existence of a convecting layer in which the mean upwelling velocity is vertically linear and proportional to an effective Rayleigh number. The dye-front experiments described above offer a systematic way to investigate both the behavior of the tuning parameter $W(\Omega)$ and also the question of how the depth of the convecting layer $h-z_c$ is determined physically. A theoretical determination of the latter is required for a complete implementation of our model; however, the suggestions outlined in section 2 can already be tested.

[28] The CAP model constitutes a new dynamical approach to modeling gravity drainage from sea ice. Using the CAP model to determine the vertical upwelling velocity w would allow a thermodynamic sea-ice model to determine, rather than prescribe, the bulk salinity profile. Such a modified sea-ice model would conserve salt as well as heat in the ice-ocean system and dynamically determine the heat capacity and thermal conductivity of the ice, the additional vertical heat transport due to convection within the ice, and net brine fluxes into the ocean.

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