EUROMECH Fluid Mechanics Fellow 2006 Paper

"ICE"

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1. Introduction

Ice is one of the most powerful agents on Earth: frost causes weathering of rocks; glacial ice sheets carve the landscape; ice is implicated in the electrification of thunderclouds; and it moderates our climate both globally and locally. The fact that we live on a partially frozen globe means that the enormous heat capacity associated with the change of phase between water and ice (it takes 80 times as much heat to melt ice as to raise the temperature of the resulting water by one degree Celsius) alone keeps us from becoming too hot or too cold. In concert with other agents, ice plays more intriguing moderating roles. Snow covered surfaces reflect 80–90% of incoming solar radiation; open sea water only 5%. The resulting ice-albedo feedback can lead to a snowball Earth or to a hot, ice-free Earth if unchecked by other processes. For example, freezing of the oceans in high latitudes increases the salinity of the surface waters, driving deep circulation of the ocean: the poleward heat transport from equatorial regions carried by the return flow, helps to check the advance of the ice cover. In this short essay, I am principally concerned with the flow of ice and flows associated with the phase change between water and ice. We shall see that fluid mechanics plays a central and often surprising role in determining the formation and demise of ice and mediating its effects in many geophysical settings.

2. Frost damage

Most of us encounter natural ice in the form of snow and frost. We are only too aware of the damage caused by frost when we see cracked flower pots, burst pipes or pot holes in our roads. The usual suspect is the well known expansion that occurs as water freezes to form ice (ice is about 10% less dense than water) but this is not the whole story nor even the main part of it.

Consider a spherical, water-filled cavity in an impermeable, rigid rock. If the temperature T is reduced to a value below 0°C then the water would like to become ice. However, in order to do so it would need to expand, which it can't in a rigid cavity. In consequence, the pressure will increase to a very high value at which the pressure-dependent freezing temperature of the water in the cavity is equal to T. The relationship between freezing temperature and pressure p is given by the Clausius-Clapeyron equation

$$L(T_m - T)/T_m = (p - p_m)\left(1/\rho_i - 1/\rho_w\right), \qquad (1)$$

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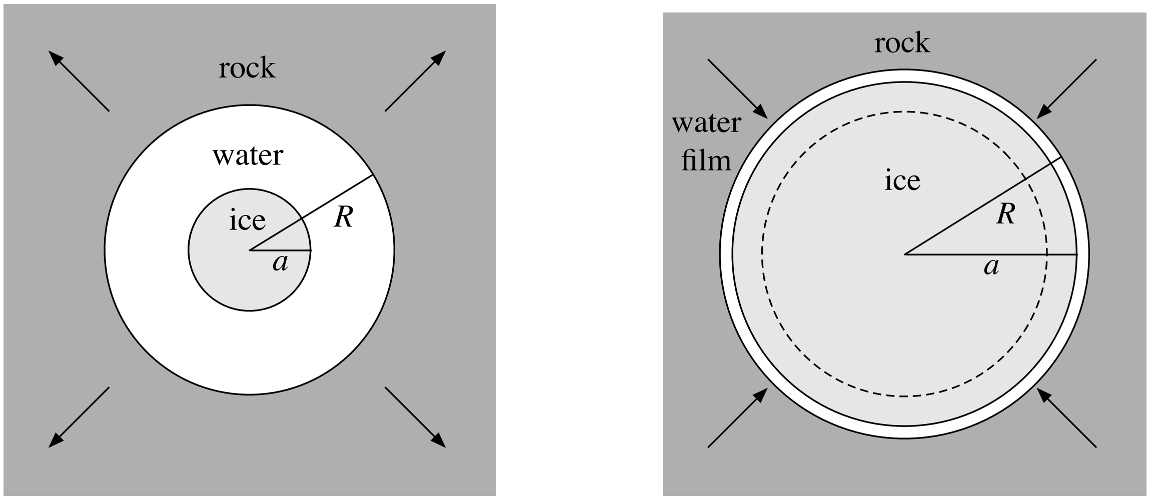

Figure 1. (a) As ice grows in a water-filled cavity in a porous rock, expansion causes water to flow out of the cavity through the rock. The pressure is elevated inside the cavity and drives the flow but has negligible influence on the elastic rock. (b) When ice fills the cavity, dispersion forces maintain a thin film of unfrozen, supercooled water between the ice and the rock. The dispersion forces pushing between ice and rock lower the water pressure in the film, which causes water to be sucked into the cavity which then expands. The dashed circle shows the initial position of the cavity wall. Relative displacements are not drawn to scale.

where T_m is the freezing temperature at reference pressure p_m ($T_m = 0°$C at $p_m = 1$ atm), ρ_i and ρ_w are the densities of ice and water, and L is the latent heat of fusion. At $-1°$C the pressure in our fictitious cavity

$$p = p^* \equiv \rho_i L(\Delta T/T_m)(\rho_w/\Delta\rho), \tag{2}$$

where $\Delta T = T_m - T$ and $\Delta\rho = \rho_w - \rho_i$, would be about 140 atm, easily enough to crack open a rock! But how did a cavity in an impermeable rock become filled with water in the first place? The rock must necessarily be permeable and that same permeability can relieve the pressure and allow ice to grow. Because of this, in most circumstances the pressure caused by expansion on freezing is wholly inadequate to deform the rock.

If the surrounding rock is modelled as a uniform porous medium, for example, then the pressure field associated with the Darcy flow caused by expansion on formation of ice in the cavity satisfies Laplace's equation, and it is readily shown that the pressure in the cavity is

$$p = (\nu\Delta\rho a^2/\Pi R)\dot{a}, \tag{3}$$

where ν is the kinematic viscosity of water, Π is the permeability of the rock, R is the radius of the cavity and a is the radius of a spherical ice formation centred in the cavity (figure 1a). To a very good approximation, the temperature field also satisfies Laplace's equation and conservation of heat at the ice–water interface is expressed by the Stefan equation

$$\rho_i L\dot{a} = -kT_r(a) = k(T - T_\infty)/a. \tag{4}$$

Equations (1), (3) and (4) can be solved for $a(t)$ but more interestingly we can use them to show that the pressure in the cavity is

$$p = p^* \, x/(x+K), \tag{5}$$

where $x = a/R$, $K = (\rho_i/\Delta\rho)^2(L/c_p T_m)(L\Pi/\nu\kappa)$, c_p is the specific heat capacity and κ is the thermal diffusivity. The highest pressure in the cavity, reached when $x = a$, is now $p^*/(1+K)$. The value of K is approximately $10^{20}\,\Pi$ m^{-2}. So in sandstones with $\Pi \approx 10^{-14} - 10^{-16}$ m^2 or limestones with $\Pi \approx 10^{-16} - 10^{-18}$ m^2, the highest pressure reached is only about $10^{-6}p^* - 10^{-2}p^*$, or at most 1 atm. Only in granites with $\Pi \approx 10^{-18} - 10^{-20}$ m^2 can the pressure become appreciable. Of course, our calculation is for a special geometry and situation but it illustrates the point that expansion often simply drives unfrozen water away from the freezing ice without a significant rise in pressure.

However, once the cavity is almost filled with ice, dispersion forces between the ice and rock molecules, mediated by those in the intervening water, act to push ice and rock apart and to keep a thin film of water unfrozen between the two. While unbalanced by elastic stresses in the rock, these dispersion forces cause the water pressure to lower in the film, which sucks more water from the surrounding saturated rock to expand the cavity (figure 1b). This process is inescapable and pushes on the rock with a pressure of about 10 atm at $-1°$C. It is this that inexorably fractures the rock. The dynamics of such fracture, which takes place in non-spherical, lenticular cavities, involves a fascinating interplay of thermodynamics, including the intermolecular dispersion forces, elastic solid mechanics and fluid mechanics [1].

3. Collapsing ice sheets

A significant proportion of the bedrock of Antarctica is below sea level. The weight of the ice sheet, several kilometers thick, ensures that it remains in contact with the bedrock inland. However, as the ice sheet flows and thins towards the coast it can eventually float on the ocean to form an ice shelf. The locus of points at which ice sheet becomes ice shelf is called the grounding line of the shelf. Significant attention is currently focused on grounding lines, particularly since recent models suggest that if the grounding line recedes to a location where the bedrock slopes downwards inland then its position will be catastrophically unstable, receding rapidly inland as the ice sheet accelerates into the ocean. The recent IPCC report contains a footnote to the effect that their predictions of sea-level rise make no allowance for the potential collapse of the ice sheets because there is currently insufficient understanding of their dynamics.

Ice sheets are typically modelled using shallow-ice models: lubrication theory with non-Newtonian (usually power-law) rheology. With the same approach, ice shelves are governed by extensional-flow equations, there being negligible tangential stress exerted on them by either atmosphere or ocean.

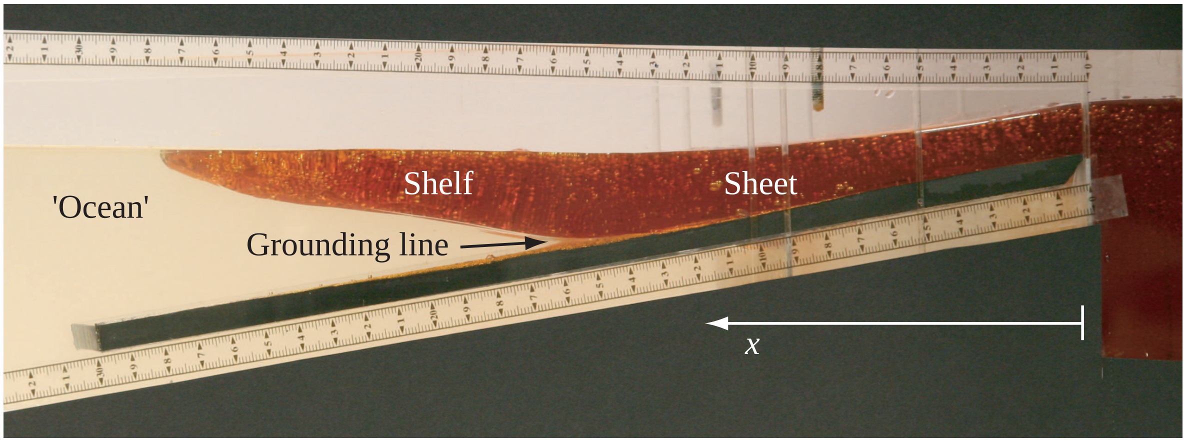

Figure 2. Photograph of an experiment in which a sheet of golden syrup flows down a slope into a denser 'ocean' of potassium carbonate solution and floats off to form a shelf. In this experiment the reservoir at the right was supplied with a constant flux of syrup. Experiment by R. Robison with H.E. Huppert and MGW.

The position of the grounding line is then a free boundary between the dynamically and mathematically distinct regions of sheet and shelf.

To explore fundamental aspects of this problem, we are conducting a series of conceptually simple laboratory experiments in which a 'sheet' of viscous fluid (golden syrup) flows down a slope into a denser 'ocean' (aqueous solution of potassium carbonate) to form a 'shelf' (figure 2). For given input flow rates, viscosities and density contrasts between 'ice' and 'ocean', we can measure the evolution of the grounding line and compare our measurements with our theoretical predictions.

Lubrication theory applied to the sheet shows that

$$h_t = -q_x = (gH^3 h_x/3\nu)_x \qquad \text{in} \qquad 0 < x < a(t), \tag{6}$$

where h is its height above sea level, H is its thickness to the bedrock, ν is its viscosity, g is the acceleration due to gravity and $a(t)$ is the horizontal position of the grounding line. At the grounding line, we apply a floatation condition

$$\rho_i g h(a) = \rho_w g' b(a), \tag{7}$$

where $g' = g(\rho_w - \rho_i)/\rho_w$ and $b(x) = H - h$ is the local depth of the ocean, and balance the depth-integrated longitudinal stress

$$4\nu(q_x + gH^2 h_x^2/2\nu) = g'H^2/2 \tag{8}$$

on either side of the grounding line (e.g. [2]). These equations combine to give an evolution equation for the grounding line

$$(b_x \rho_w g'/\rho_i g - h_x)\dot{a} = gH^2 h_x^2/2\nu - g'H^2/8\nu. \tag{9}$$

If the sheet is supplied by a constant flux q_0 upstream then the grounding line reaches a steady position

$$a = (\rho_i/\rho_w)(6\nu q_0/g)^{1/3}(g/g')^{1/6}/b_x. \tag{10}$$

This combined theoretical and experimental approach is allowing us to test fundamental aspects of the theoretical modelling such as the balance of longitudinal stress (equation (8)), where a lot of current research and debate is focused.

4. Ice in the ocean

Heat transfer from ocean to atmosphere in polar regions has two potential effects on the local density of the ocean: if the water is above its freezing temperature then it will cool and become denser; if at its freezing temperature then ice will form and the remaining water will become more saline and hence more dense without appreciable change in temperature. For a given heat flux F to the atmosphere, the buoyancy fluxes resulting from cooling and freezing are respectively

$$B_C = \alpha g F / c_p \qquad \text{and} \qquad B_F = \beta S_0 g F / L, \tag{11}$$

where α and β are the linear density coefficients for temperature and salinity respectively, and S_0 is the salinity of the ocean. The ratio of these buoyancy fluxes

$$B_F / B_C = \beta S_0 c_p / \alpha L \approx 10 - 20 \tag{12}$$

given values typical of the polar oceans. These simple considerations show that the highest buoyancy fluxes in polar oceans occur when there is simultaneously a high heat flux and ice formation. Such conditions are maintained in polynyas, for example, where newly formed ice crystals are blown by strong winds so that the relatively warm ocean is continually exposed to the cold atmosphere. Antarctic polynyas are responsible for the world's densest, most saline abyssal waters. Similarly high buoyancy fluxes also occur in marginal ice zones and during the initial refreezing of leads. If the summer-time extent of Arctic sea ice continues to recede then the Arctic Ocean will be characterized much more by thin, first-year sea ice, with a consequent increase in the importance of salt-driven convection.

The salt flux (hence buoyancy flux) associated with freezing of the oceans is much more complicated to assess once a layer of consolidated sea ice has formed. Sea ice is a mushy layer [3, 4], a reactive porous medium of pure ice crystals bathed in concentrated brine. Whether and how quickly that brine can drain into the oceans depends on intricate physical interactions between fluid flow and phase change in the interior of the sea ice. In particular, flow from cooler to warmer regions of sea ice causes the ice crystals that form its matrix to dissolve. That increases the local permeability and the flow, which becomes focused into narrow brine channels (figure 3). The fluid dynamics of this process is governed principally by a Rayleigh number

$$R_m = (1 + L / c_p m S_0)\, \beta g \Delta S \Pi h / \kappa \nu, \tag{13}$$

where m is the slope of the freezing temperature variation with salinity. This Rayleigh number, which is characteristic of convection in mushy layers, reflects

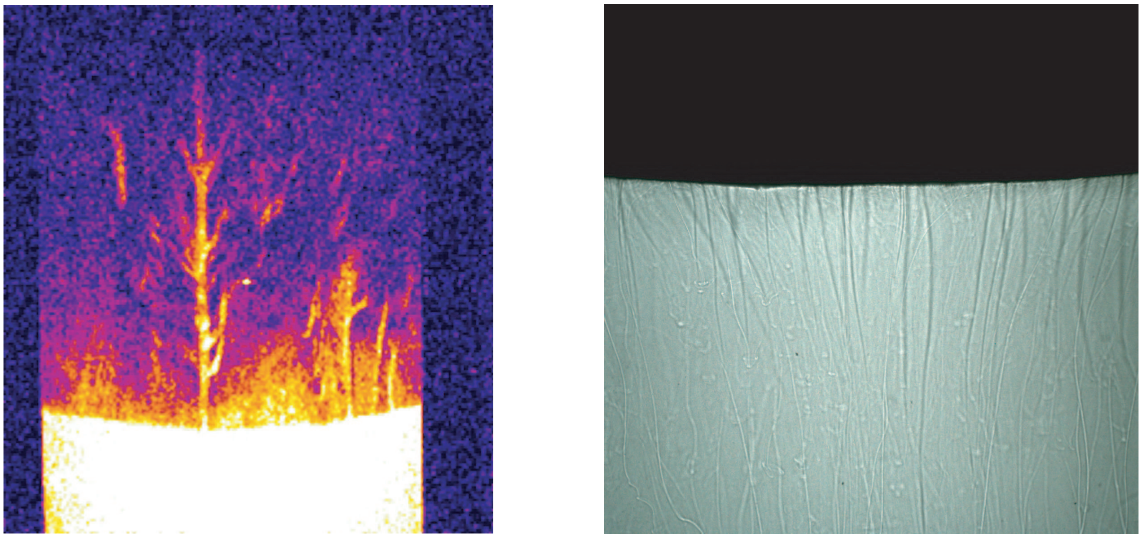

Figure 3. (a) MRI image of the interior of a convecting mushy layer showing a large vertical dissolution channel with side branches and a number of smaller channels [5]. (b) Shadowgraph image of plumes of brine emanating from brine channels in laboratory grown sea ice [6].

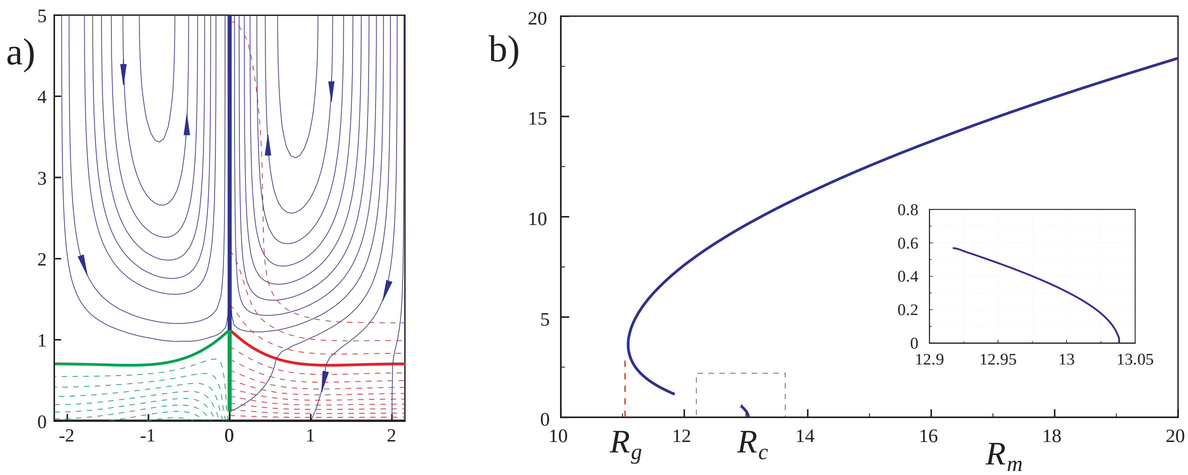

Figure 4. (a) Streamlines (thin solid curves), isotherms (dashed curves on right) and contours of solid fraction (dashed curves on left) calculated for steady solidification of a binary alloy [7]. The thick solid curve shows the interface between the mushy layer (below) and liquid region (above). Liquid flows through the mushy layer, some of it returning via a chimney (thick vertical line) in the mushy layer to emerge as a plume in the liquid region. (b) A measure of the strength of the convective flow as a function of the Rayleigh number R_m. The subcritical bifurcation to weak convection in the mushy layer from the linear critical point R_c is shown enlarged in the inset. The upper branch relates to states in which convection causes dissolution channels (chimneys) to form in the mushy layer, as shown in (a). The minimum Rayleigh number at which steady convection can occur is given by R_g.

the facts that the flow is in a porous medium of permeability Π, that the buoyancy is dominated by salinity variations ΔS, and that the dissipation of that buoyancy is effected by phase change mediated by the thermal field with diffusivity κ. The prefactor $(1 + L/c_p m S_0)$ reflects the fact that the effective heat capacity of mushy layers is dominated by the internal release or absorption of latent heat.

Detailed theoretical and numerical analyses have been made of convection in mushy layers (figure 4) and many of their properties have been verified experimentally [8]. However, it remains a challenge to develop a dynamical model of the salt (buoyancy) fluxes from sea ice simple and robust enough to be incorporated into large-scale climate models.

5. Conclusion

We have seen that ice plays a significant role in many environmental processes and is of great interest to engineers, geoscientists and physicists. It is also of great importance in biology (ice algae account for more than half of Arctic marine primary production), medicine (for cryo-preservation of cells and tissue) and chemistry (some ozone-destroying aerosols originate from frost flowers on sea ice) to give but a few examples. And for the applied mathematician and fluid dynamicist, the study of ice involves interesting challenges in free-boundary problems and diverse nonlinear interactions between flow and structure.

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