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EXPERIMENTAL AND THEORETICAL MODELLING OF ICE SHEET–SHELF GROUNDING LINES

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Summary We have used simple laboratory experiments with viscous fluids to explore the dynamics of grounding lines between Antarctic ice sheets and the freely floating ice shelves into which they develop. Ice sheets are shear-dominated gravity currents, while ice shelves are extensional gravity currents having zero shear to leading order. Though ice sheets have non-Newtonian rheology, fundamental aspects of their flow can be explored with Newtonian fluid mechanics. We have derived a mathematical model of this flow that incorporates a new dynamic boundary condition for the position of the grounding line, where the gravity current loses contact with the solid base. Good agreement between our theoretical predictions and experimental measurements gives confidence in the fundamental assumptions of our model, which can be incorporated into shallow-ice models to make important predictions regarding the dynamical stability of shelving ice sheets.

INTRODUCTION

The Antarctic ice cap contains several tens of millions of cubic kilometers of frozen water, enough to raise sea level by 60–70 metres should it melt. So, while sea level is estimated to rise by a few tens of centimetres in the next hundred years by direct melting of the polar ice caps due to global warming, a dynamical collapse of the ice sheets has the potential for a truly catastrophic rise in sea level. The recent IPCC report demurs from making any quantitative prediction of such a rise because there is currently insufficient confidence in our ability to model the dynamics of ice sheets.

Most of the bedrock of Western Antarctica is below sea level and it is simply the weight of the ice sheet, some 2–3 kilometers thick, that keeps it grounded. However, the sheet thins as it flows from the continental interior, and there comes a point at which the ice can float freely on the ocean and it detaches from the bedrock to become an ice shelf. The locus of such points is called the grounding line of the ice shelf. Its position is determined in part by the floatation condition just described but that is not sufficient. For example, a condition of continuous longitudinal stress has been added to formulate closed mathematical models of ice sheets dominated by basal shear terminating at grounding lines with free ice shelves [1, 2]. However, the predictions of these models are untested and the characteristics of the flows considered are difficult to reproduce experimentally.

EXPERIMENTS

To explore fundamental aspects of this problem, we have conducted a series of conceptually simple laboratory experiments in which a ‘sheet’ of viscous fluid (golden syrup) flows down a slope into a denser ‘ocean’ (aqueous solution of potassium carbonate) to form a ‘shelf’ (figure 1). These Newtonian fluids can be easily handled and characterized in the laboratory, and their flows are well described by the Navier-Stokes equations. For given input flow rates, viscosities and density contrasts between ‘ice’ and ‘ocean’, we can measure the evolution of the grounding line and compare our measurements with our theoretical predictions.

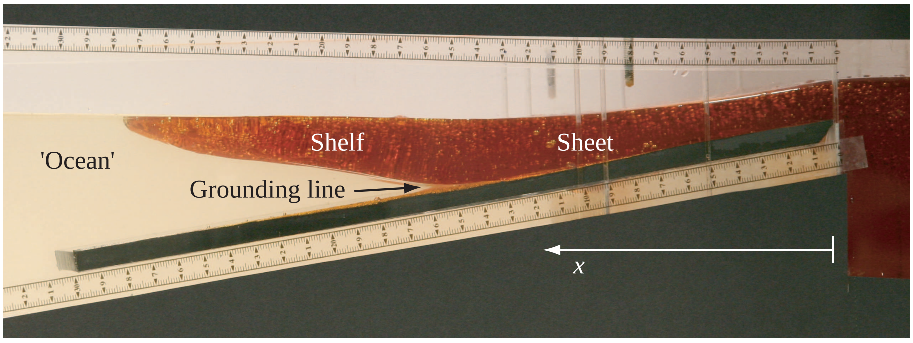

Figure 1. Photograph of an experiment in which a sheet of golden syrup flows down a slope into a denser ‘ocean’ of potassium carbonate solution and floats off to form a shelf. In this experiment the reservoir at the right was supplied with a constant flux of syrup.

THEORY

Consider a two-dimensional, grounded, viscous gravity current (sheet) developing into an extensional surface gravity current (shelf), as depicted in figure 2. We seek a description of this flow within the approximations of lubrication (thin-film) theory.

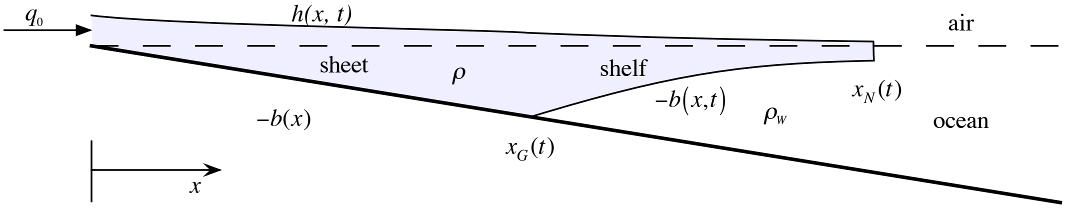

Figure 2. Schematic of the sheet–shelf system defining variables used in the theory.

The shelf in $x_G(t) < x < x_N(t)$ is assumed to be in local hydrostatic equilibrium so that

$$\rho H = \rho_w(H - h) \quad \Rightarrow \quad h = H(\rho_w - \rho)/\rho_w, \tag{1}$$

where $H = h + b$ is the total depth of the shelf. We consider the case in which the fluids bounding the shelf (air above and water below) exert no tangential stress on it. Therefore, to leading order in lubrication theory, the horizontal velocity $u(x, t)$ is independent of depth and satisfies the extensional flow and continuity equations

$$(4\mu/H)(Hu_x)_x = \rho g h_x = \rho g' H_x, \qquad H_t + (Hu)_x = 0, \tag{2,3}$$

where $g' = g(\rho_w - \rho)/\rho_w$. This system of equations can be integrated using characteristics to determine $H(x, t)$ in terms of the time history of the position of the grounding line and the thickness and volume flux of the sheet there.

Lubrication theory applied to the sheet in $0 < x < x_G(t)$ gives

$$h_t = -q_x = (gH^3 h_x/3\nu)_x \qquad \text{where} \qquad H = h + b. \tag{4}$$

At the grounding line, we apply the floatation condition (1) and balance the depth-integrated longitudinal stress to determine, after some lengthy algebra, an evolution equation for the grounding line, which for shallow slopes is given by

$$(b_x \rho_w g'/\rho_i g - h_x)\dot{x}_G = gH^2 h_x^2/2\nu - g'H^2/8\nu. \tag{5}$$

If the sheet is supplied by a constant flux q_0 upstream then we can show that the grounding line reaches a steady position (cf [1])

$$x_G = (\rho_i/\rho_w)(6\nu q_0/g)^{1/3}(g/g')^{1/6}/b_x. \tag{6}$$

RESULTS AND CONCLUSIONS

Some results of our experiments and theory are shown in figure 3. There is a good trend between the theoretical and experimental results, and the discrepancies may be explainable in terms of viscous stresses exerted by the side walls of the tank and by the addition of fluid to the tank. These effects are currently being quantified. This is the first experimental study of grounding lines and our theoretical approach has provided an explicit differential equation governing their evolution. This approach will lead eventually to robust and confident predictions of the future of our polar ice caps.

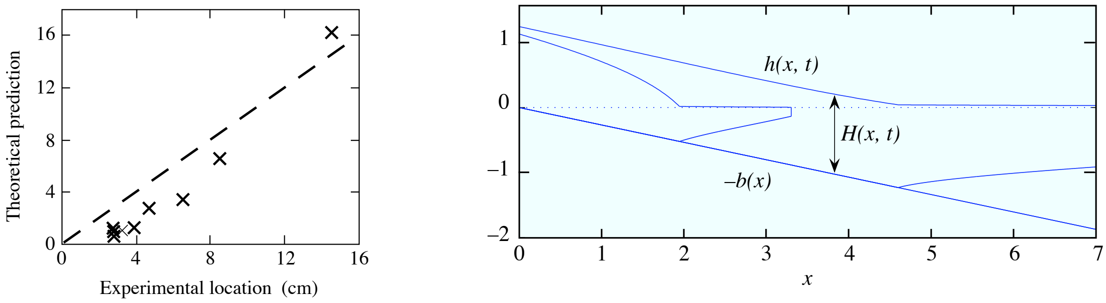

Figure 3. (a) Comparison between the measured position of the grounding line and the theoretical prediction (6). (b) Numerical solutions of the governing equations (1)–(5) at two different times, showing the qualitative evolution of the sheet–shelf system.

References

[1] Wilchinsky, A.V. & Chugunov, V.A.: Ice-stream–ice-shelf transition: theoretical analysis of two-dimensional flow. *Annal. Glac.* **30**: 153–162, 2000.

[2] Schoof, C.: Marine ice-sheet dynamics. Part 1. The case of rapid sliding. *J. Fluid Mech.* **573**: 27–55, 2007.