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Comments on 'On Convective Style and Vigor in Sheet-like Magma Chambers' by Bruce D. Marsh

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In a recent paper in this Journal, Marsh (1989*b*) put forward a series of incorrect statements about the fluid mechanics of crystallizing convective systems and arrived at conclusions which claim to change the way in which one should describe convection in magma chambers. The theoretical ideas used to support these conclusions are inconsistent with well-established and tested fluid dynamical principles. Along with many of our fluid dynamical colleagues we are concerned about the influence that such inappropriate theory by a well-known geologist will have. We have previously set out our point of view in a series of publications, some of which will be referred to below, but we are constantly approached by geologists to explain our explicit objections to Marsh's published statements in a way that all can read and consider for themselves. We do so briefly here, confining ourselves *entirely* to questions of fundamental fluid mechanical principles. Geological observations or their interpretation are irrelevant in the discussion of these points as no explanation of a geological feature based on incorrect physical arguments can be of any value.

Marsh's thesis, put forward in three separate publications (Marsh, 1988, 1989*a*, 1989*b*), is that the convective motion in almost all naturally occurring magma chambers is weak or non-existent even when the contained magma is still quite fluid. His arguments are based on three false premises. First, that 'the characteristic length scale of the governing Rayleigh numbers, which is time dependent, is the sublayer thickness' (abstract, p. 479 of the 1989*b* publication). (It is true, but irrelevant, that this length scale, which is an internal scale determined by the conductive cooling of the fluid through the roof, is very much less than the overall height of a typical magma chamber.) Second, that the constraint that heat can only be transferred through the surrounding country rock by conduction necessitates that convection within the chamber is suppressed. Third, that the absence of any superheat in a magma makes it impossible to sustain convection except in the very earliest stages of cooling. We shall briefly address each of these points in turn, together with several other misconceptions which are directly related to them.

First, it is a well-established fluid mechanical result that in the interior of a fluid, the whole depth is relevant in determining the Rayleigh number and the 'vigor of the flow', i.e., the velocities which are generated by a given boundary layer heat flux; and for deep chambers this Rayleigh number will be large. It is not surprising that the total depth matters in determining the convective velocities. The turbulent motions in the interior of the fluid are produced by gravity continuing to act on the dense, negatively buoyant fluid after it has broken away from the top boundary, and the motion increases in strength as the depth increases. There is an analogy here with the fact that the velocity and kinetic energy of a large stone dropped from a tall building depends on the height of the fall. The concluding summary on p. 499 thus runs counter to all previous work on convection by fluid dynamicists. Temperature-dependent viscosity, and the existence of crystallization, do not substantially change this picture, as discussed further below.

We turn now to Marsh's second notion that convection cannot possibly occur within a magma chamber because of the relatively slow conductive heat transfer through the surrounding country rocks. First, it is clear that the heat transfer through the boundaries of a fluid is directly related to the difference in temperature between the interior and the solid roof or floor. This means that the temperature difference driving the convective motions in the fluid can change as a result of either the change in the conductive heat transfer to the overlying solid or the slow evolution of the temperature in the interior of the fluid; and in some cases as a result of both of these effects acting simultaneously. In all these cases, it is only the details of this slow evolution which constitute the difference between the continuously heated and cooled case, and cooling at the top boundary only. It is true that the heat flux out of a magma chamber is dominantly controlled by conduction through the roof wall rocks. This flux will change in time as the temperature in the magma decreases but, as demonstrated in detail by Turner *et al.* (1986), it must at all times equal the sum of the flux from the fluid interior plus that supplied by latent heat due to crystallization at the roof. For the non-crystallizing case, Huppert & Sparks (1988) present a completely rigorous solution which shows, in contradiction to Marsh's assertions, that a conductive flux through the boundary can balance a *turbulent convective flux* out of the interior. A more heuristic, but nevertheless totally correct, argument which came to the same conclusion, was independently presented by Carrigan (1987). The important physical point here is that most of the temperature drop between the interior of the fluid and the solid far from the solid/fluid boundary occurs in the solid itself. A very much smaller temperature difference ΔT acts within the fluid, to produce the intermittent breaking away of convective elements in the unstable boundary layer. This idea is expanded in the next paragraph.

Even with a very small effective ΔT, which might be only a fraction of a degree centigrade in some cases (Martin *et al.*, 1987; Worster *et al.*, 1990), the thickness d of the unstable boundary layer predicted for a typical magma chamber is of the order of a few metres, very much less than the total depth D. The corresponding Rayleigh number of the interior convection can still be large, as the small ΔT multiplies the cube of the large depth in the evaluation of the Rayleigh number. The boundary layers become unstable on their own length scale d and break away from the boundary as if the opposite boundary were not there. Thus the heat flux (but, as we have indicated above, *not* the vigour of convection in the interior of the chamber) *is* independent of D. It should be emphasized (see also Turner, 1973, p. 213) that the independence of heat flux on D is a high Rayleigh number limit. This result also implies that the Nusselt number, defined in the usual way as the ratio of the actual convective heat flux due to a given temperature difference *between the solid boundary and the interior of the fluid* to the flux produced by molecular conduction with the same temperature difference acting over the whole depth of the convecting part, is given, for constant viscosity, by $\mathrm{Nu} = c\mathrm{Ra}^{1/3} = D/d$, where c is $\sim 0{\cdot}1$. This Nusselt number is *always* greater than one; and it will generally be large, of order 10^3 or greater, in magma chambers. Marsh defines, on pp. 505 and 518, 'Nusselt numbers' which include the temperature difference *in the solid wall* and as justification incorrectly cites Homsy (1973), who considered a fluid layer *only*. Marsh thereby calculates values of his 'Nusselt numbers' which are less than unity, and 'deduces' correspondingly small values of Ra. It is misleading to introduce new parameters and call them by established names and thereby to suggest incorrectly that convection is weak.

We now consider the possible effect of a temperature-dependent viscosity on the above conclusions. We agree with Marsh's statement in the last full paragraph of p. 495, and the preceding arguments showing that convection occurs in a layer of nearly constant viscosity, with an immobile viscous layer above it through which the heat flux occurs by conduction alone. The temperature difference across the unstable part of the boundary layer is so small that it has negligible effect on viscosity. It is only the temperature difference and the associated density differences which affect the convection in the fluid below and *this acts on the full depth of that part of the fluid which is at constant viscosity*, just as it would if there were a solid boundary above. As shown by Martin *et al.* (1987) and Worster *et al.* (1990), the temperature difference between the immobile (solid or highly viscous) roof and the fluid interior is typically a few tenths of a degree centigrade. This will produce an insignificant variation of viscosity across the whole convecting region. That is, the variable viscosity changes the thickness of the immobile, conductive region, but it does not change the nature of the flow outside it. Marsh (Figs. 5 and 8) arbitrarily picks a length scale d', corresponding to the lower-viscosity outer part of the boundary layer, to characterize the 'mobile' part of the flow, but in fact the whole of the fluid below this also has a low viscosity and will take part in the convection. A similar argument applies when considering the effect of crystallization and growth of a solid or mushy zone at the upper boundary. At a very early stage of cooling of a highly superheated chamber the growing crystal boundary can advance so fast that it overtakes the fluid motions and prevents any convection. For any realistic conditions in a magma chamber, however, this stage is soon past, and the stability criterion at the solid–fluid interface is essentially the same as it is at a stationary boundary. That is, the convective motions in the fluid below the crystallizing roof will depend again on the temperature difference across the *fluid* boundary layer, and on the whole depth of the chamber.

To address the third point, the absence of superheat during a large fraction of the cooling history of a magma chamber, we summarize the results set out in a recent paper by Kerr *et al.* (1990). They have calculated the effect on melts of cooling from above with the kinetics of crystallization taken into account as first suggested by Brandeis & Jaupart (1986). Even without any initial superheat whatsoever, both laboratory experiments and related theory demonstrate that convection develops and allows substantial internal cooling, crystallization, and differentiation to occur. The composition of the melt evolves as a result of crystallization on the floor in such a way that the liquidus temperature decreases with time. A recent paper by Worster *et al.* (1990), based on these concepts but using compositions in the An–Di system, concluded that the interior of a magma chamber can remain always at the evolving liquidus, while thermal convection is driven by a small degree of supercooling (of a few tenths of a degree centigrade) at the top of the chamber produced by the heat flux through the roof. The relative amount of top and bottom crystallization depends on the depth of the chamber; in thin sills the predominant growth is that of a crust at the roof, whereas in large magma bodies crystallization at the floor becomes increasingly important, as discussed by Sparks (1990).

In summing up, we wish to emphasize three points. First, although the fluid being considered is crystallizing magma with temperature-dependent viscosity, these differences from a simple fluid system modify, but do not entirely change, the firmly based physical principles on which previous theoretical and experimental work on convection at high Rayleigh numbers is based. Second, we regard it as a most welcome (and important) development that dynamical concepts have begun to be introduced more widely and taken seriously in many geological contexts. It is essential, however, in order to make progress, to apply the correct concepts. Interested parties are advised to work through the theoretical arguments for themselves, or consult an experienced fluid dynamical colleague, before accepting or applying results which make claims that are not supported by clear physical reasoning or detailed theory. We conclude with a very general, third point. In a typical magma chamber, possibly many hundreds of metres high, there will be thermal differences due to contact with relatively cool surrounding country rock, compositional differences due to crystallization, and possibly even differences in gas content due to volatile exsolution. The existence of such density perturbations in a large body of fluid would make any experienced fluid dynamicist confident that strong convective motions are highly probable, and definitely *not* out of the question.

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