HEAT AND MASS TRANSFER IN GEOLOGY AND GEOPHYSICS

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ABSTRACT

Some of the fluid mechanical problems of relevance to the Earth sciences are briefly reviewed in this article. Many of the situations considered involve applications of the fundamental concepts of heat and mass transfer. Additionally, there are many exciting problems suggested by geology and geophysics which require new ideas and approaches to be developed.

1. INTRODUCTION

The Earth upon which we live provides a vast array of situations in which both heat and mass transfer play a fundamental role. Numerous problems arise which require new areas of the subject to be investigated. In addition, there are many interesting problems in the earth sciences whose solution involves the application of concepts and results which have already been derived in the heat and mass transfer literature. Such applications are often to situations whose length and time scales differ greatly from those for which the theory has previously been used. Comparison of the theoretical, and in many cases also experimental, results with data from field observations adds credence to the fundamental concepts as well as providing useful illustrative examples.

Just a very few of these problems are reviewed in this article. The presentation will concentrate on highlighting the fundamental ideas in some of the areas of research that I and my group have been investigating recently. I trust that this bias towards my own research interests will be understood by the general reader. Most of the details of the investigations will be omitted and the interested reader will be referred to the original articles.

The review will commence with a new model for the thermal evolution of the solid inner core of the Earth, which is currently being put forward with my colleagues Bruce Buffett, John Lister and Andy Woods (Buffett *et al.*, 1991). We are suggesting that the numerous and complicated processes that determine the solidification and rate of growth of the inner core can be quite simply modelled to indicate that the inner core has been in existence for 2×10^9 yr, compared with the age of the earth of 4.5×10^9 yr, and that the time taken for the core to totally solidify will be almost an order of magnitude longer.

The third section of the paper briefly reviews the generation of magma, or molten rock, within the Earth and the processes which either lead to, or are consequent upon, volcanic eruptions. This new and extensive range of scientific problems has become known as *geological fluid mechanics*, and is generating much excitement in a number of research groups throughout the world. In addition, its progress is being keenly watched by a large number of traditionally trained geologists who desire to know if this new and quantitative approach to their subject will lead to a fundamentally different way of approaching problems.

The fundamental fluid mechanics involved in melting the roof of a container by an entrapped hot turbulent flow is described in the next section. There are three distinct possibilities dependent on the density of the melt relative to that of the underlying fluid. If the melt is relatively heavy it sinks and can mix with the hot fluid. If relatively light it forms a separate layer between the solid and the hot fluid. An intermediate case is also possible, and has been photographed, wherein the melt is initially heavy and sinks into the hot fluid whereupon thermal transfers reverse the buoyancy and the sinking melt slows down and eventually rises.

The turbulent flow of a hot fluid over a bed and the subsequent melting of the bed is then briefly described. Both the initial value problem and the steady-state response is discussed. The work has important applications to the turbulent flow of relatively hot and inviscid lavas over the surface of the Earth many years ago.

2. THE EVOLUTION OF THE INNER CORE OF THE EARTH

The Earth can be divided into three regions: a solid inner core, whose current radius is 1221 km; a liquid outer core, which extends to 3480 km; and a solid mantle. Concomitant with heat being lost from all regions of the earth, the fluid outer core gradually solidifies to extend the radius of the inner core which is made up of almost pure iron. The residual fluid released by the solidification is poor in iron and is thereby less dense than the original melt. This compositional difference, along with thermal differences across the outer core, drives vigorous convection in the liquid outer core at a Rayleigh number which is extremely large, and may even exceed 10^{25}. The motion in the outer core is influenced by the rotation of the earth and, because the fluid in the outer core is a good conductor, it is also influenced by electromagnetic effects. Indeed, it is generally believed that the all-important external magnetic field of the earth is maintained by the interaction of the convection within the rotating outer core and the magnetic fields which permeate it. As described, this is a very complicated, highly nonlinear coupled system, concerning which there have been a vast number of intensive studies over many decades. Our new approach, which is based on global heat conservation, quantifies the dominant features of the thermal evolution of the core without requiring a detailed knowledge of the convective processes involved. Our model is constructed using the idea that the convection in the outer core is sufficiently strong to keep the fluid there well mixed. Because of the slow growth of the inner core, its surface will be in thermodynamic equilibrium with the (spatially uniform) temperature, say $T(t)$, of the much more voluminous outer core. This temperature will be identical to the (prescribed) solidification temperature of the inner core, which is a function of the radius, say r_i, of the inner core.

The heat extracted from the cooling outer core is transported through the overlying mantle to the surface of the Earth by thermal convection. The relatively sluggish and more massive mantle controls the cooling of the core by regulating the heat flux, say $f_m(t)$, across the core-mantle boundary. While the magnitude and time dependence of $f_m(t)$ will depend on the details of mantle convection, in order to focus on the dominant processes involved, we can simply prescribe $f_m(t)$ and investigate the consequent thermal evolution of the core. In spherical geometry, conservation of heat may be expressed as

$$\frac{4\pi}{3}(r_0^3 - r_i^3)c\frac{dT}{dt} - 4\pi r_i^2 L\frac{dr_i}{dt} = 4\pi r_i^2 f_i(t) - 4\pi r_0^2 f_m(t), \tag{2.1}$$

where c and L are the assumed constant specific heat and latent heat per unit volume and r_0 is the constant radius of the fluid outer core. The conductive heat flux f_i from the inner core, which is given by Fourier's law, is determined from the solution of the thermal diffusion equation in the inner core. Two limiting cases immediately present themselves. These correspond physically to the limits of a perfectly insulating and a perfectly conducting inner core. The difference between these two extremes can be shown to be of order $(r_i/r_0)^3$, which is currently negligible. We hence restrict attention to the simpler case of a perfectly insulating inner core, in which case no heat is conducted from the inner core, and $f_i \equiv 0$. A first integral of (2.1) can then be written as

$$c\int_0^{r_i(t)} \frac{dT}{dr}(r_0^3 - r^3)dr - Lc^3 = -3r_0^2 \int_0^t f_m(t)dt. \tag{2.2}$$

Over the range of pressures in the core, the solidification temperature is a function of pressure which, by using the hydrostatic equation and Newton's law of gravitation, can be shown to be a quadratic function of the radius from the centre of the Earth. Thus we can write

$$T(r) = T(0) - Ar^2, \tag{2.3}$$

where A is a (fairly well) known constant. Inserting (2.3) into (2.2) and carrying out the resulting integrations, we find that

$$\eta^2 + S\eta^3 = B\int_0^t f_m(t)dt, \tag{2.4}$$

where

$$\eta = r_i/r_0, \quad B = 3/(Acr_0^3) \tag{2.5a,b}$$

and the Stefan number

$$S = \tfrac{1}{3} BLr_0. \tag{2.5c}$$

Using currently accepted values of physical parameters of the Earth, $S \sim 0.4$ and since $\eta \sim 1/3$ the second term in (2.4) is significantly smaller than the first. This indicates that at the moment the growth of the inner core is almost entirely controlled by a balance between the cooling of the outer core and the heat flux into the base of the mantle; the latent heat released from the solidification of the inner core and the heat transfer from the inner core play a secondary role.

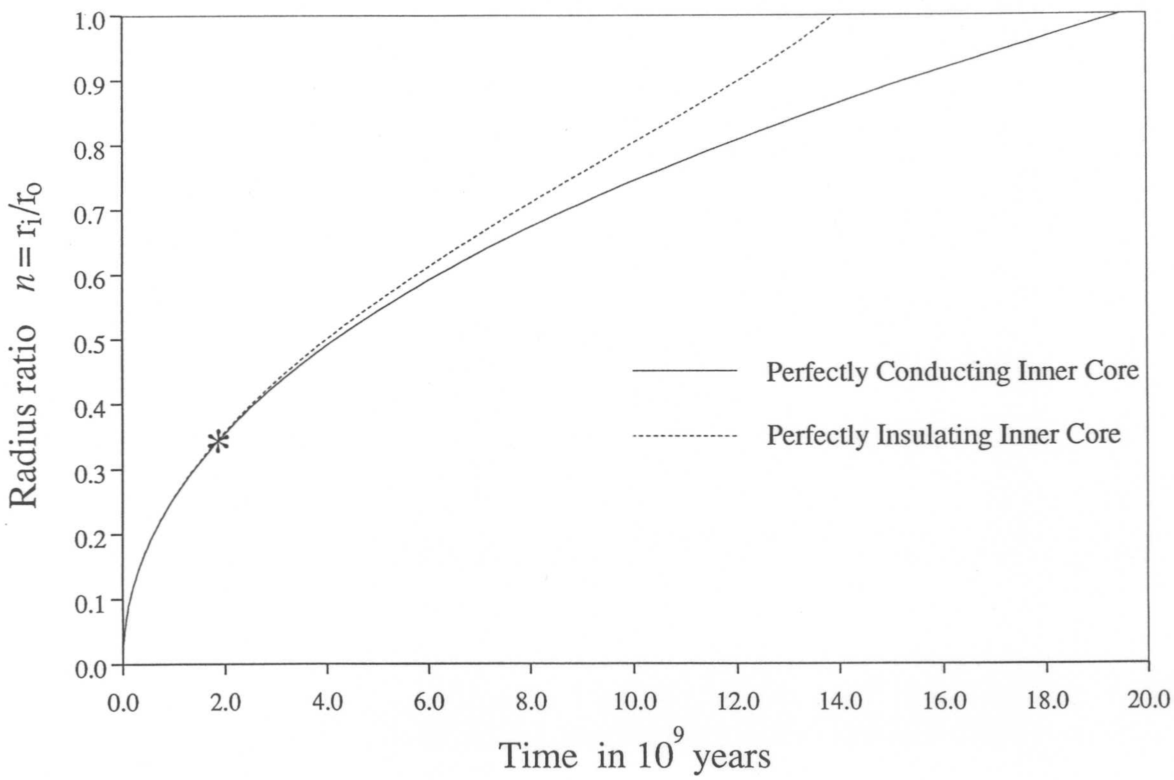

FIGURE 1. The ratio of the radius of the solid inner core to the outer core, η, as a function of the time since the initiation of the solid inner core. The current values are denoted by a $*$.

Figure 1 presents the nondimensional radius of the inner core as a function of time, as determined both from (2.4) and from the analogous relationship obtained on assuming that the inner core is a perfect conductor (in which case its temperature would be spatially uniform). At the present time, marked with a $*$, there is very little difference between the two curves. Not until the final stages of the solidification, in approximately another 10^{10} years, will there be a significant difference.

3. HEAT AND MASS TRANSFER IN MAGMAS

The mantle of the Earth, whose volume is approximately $7/8$ of the total, is predominantly solid. However, a large number of important fluid processes occur within various regions of the mantle. It is believed by some that large pools of fluid in the form of molten rock, or magma, can be found at the base of the mantle, in the so-called D'' layer. Much of this melt is less dense than the surrounding rock and so, in this scenario, fluid rises in the form of mantle plumes all the way to the surface of the Earth. (See Loper, 1990 for a more complete discussion of the fluid processes involved and the associated geophysical observations.)

Other sources of melt within the mantle are better documented. Large scale tectonic motion of the earth causes solid rock to flow slowly into regions where the local temperature exceeds the melting temperature of some of the many components that make up the rock. These melts are again generally less dense than the surrounding solid rock and migrate upwards along the interconnected grain boundaries. The motion is driven by the pressure due to the overlying rock matrix which compacts as the melt rises. This process of compaction, made famous by arguably the leading geophysicist in the world, Dan McKenzie, in 1984 plays an essential role beneath mid-ocean ridges and at sites where the upper mantle is stretched. The melt produced in this way can lead to outpourings on the surface of many millions of cubic kilometers, as in the Deccan traps of India (White and McKenzie, 1989a,b).

The flow of melt from its source by compaction is an intricate fluid mechanical process of which it is said that thermodynamics plays no role, though I have my doubts about the veracity of this statement. At some stage the melt finds its way into a magma chamber which acts as a storage reservoir for the liquid rock. These magma chambers can range in size from being rather small to containing up to a few tens of thousands of cubic kilometers of partially liquid rock. Within a magma chamber many fluid mechanical and thermodynamic process can occur.

Due to the transfer of heat to the surrounding medium, known as the country rock – which is not a new style of American western music – the magma may partially or even totally solidify. Being a multicomponent fluid with different solidification temperatures for each component, the minerals solidify in a fixed order, leaving behind at any time a melt whose composition differs from the original. By this important process, known as *convective fractionation*, the composition of both the solid product and the residual magma changes with time and hence also spatially. The change in composition of the liquid as solidification proceeds can drive vigorous compositional convection, often enhancing the thermal buoyancy maintained across the chamber.

Alternatively, magma of one composition may be intruded into the chamber at a temperature which exceeds the melting temperature of the surrounding country rock of a difference composition. The rock will melt and, depending on its density, will either pond at the site of melting or flow into the original magma. The continental crust of the Earth is reworked by this process to produce many of the granites we see on the surface of the Earth. Some aspects of the fundamental fluid mechanics and the associated heat and mass transfer have been investigated (Huppert and Sparks, 1988 a,b,c) and will be reviewed in section 4.

Due to either large-scale tectonic motions or fluid dynamical process which occur within the magma chamber, the pressure increases and exceeds that which the overlying rock can sustain. The rock is then torn apart and a volcanic eruption can follow, in which the magma either makes a new conduit from the chamber to the surface or uses a pre-existing conduit. The multiphase system of rock fragments,liquid rock, exsolved gas and fine ash particles can ascend at a speed of a few hundreds of metres per second to erupt into the atmosphere to form a volcanic eruption column. This column has been known in current times to penetrate up to 45 km into the atmosphere. The motion is driven only to a small extent by the exit momentum and mainly by the heat transfer from the small, very hot ash particles (at a temperature of approximately $1000°C$) contained in the plume to the entrained ambient air. This makes the air significantly less dense than the surrounding atmosphere and the negative buoyancy generated this way is the main driving mechanism (Woods, 1989).

Lava flows, of up to a few tens of cubic kilometers, are also frequently associated with eruptions. They slowly solidify at the top and at the base as they lose heat to the atmosphere and (to a lesser extent) to the ground over which they flow. Many of these situations are described more fully in Huppert (1986). In addition, some of these processes are contained in an artist's sketch of a typical cross-section through a volcano which is presented in figure 2.

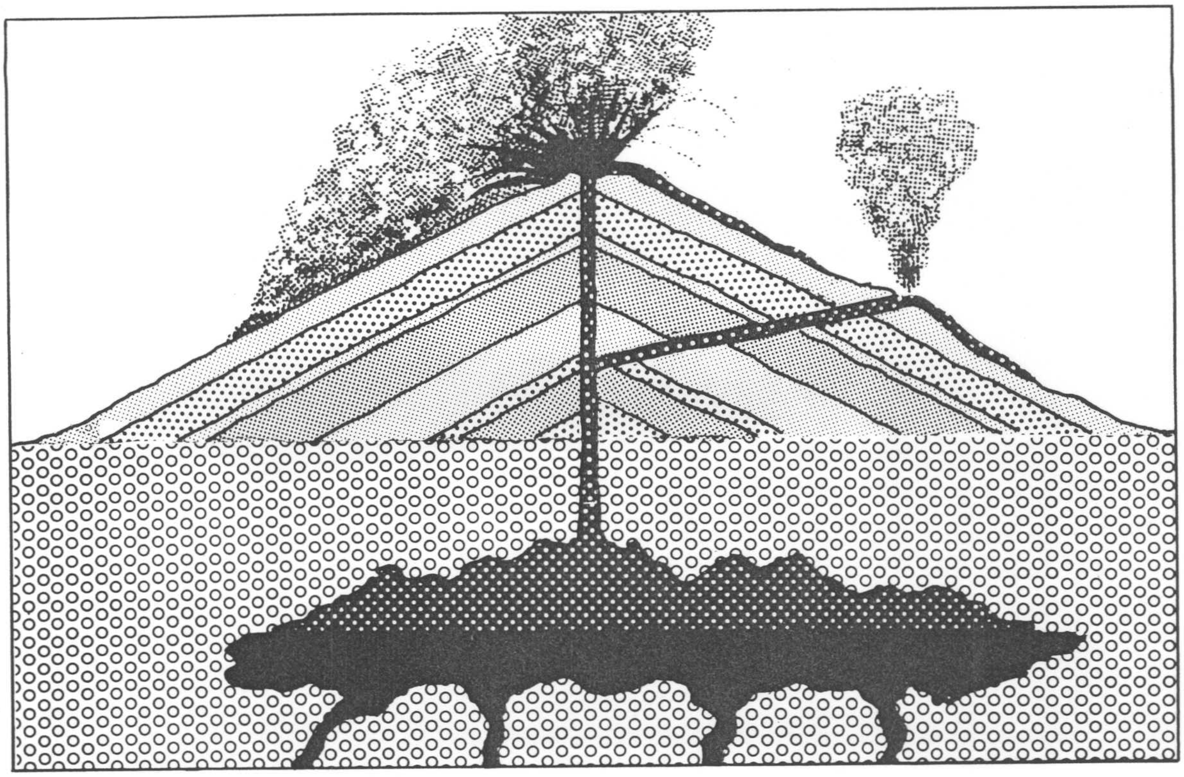

FIGURE 2. A pictorial sketch of a typical cross-section through a volcano and connected magma chamber. Magma flows from depth into the stratified chamber, where it may reside and cool for some time, before coming to the surface and being erupted. Plumes rise from the vents which also discharge lava and heavy, ash-laden gravity currents.

4. MELTING THE ROOF OF A CONTAINER

There are numerous situations, in both geology and industry, in which the walls of a container melt due to the input of a hot fluid. The melting may occur at the roof or the base of the container or along the predominantly vertical side walls. Each of these will involve different fluid mechanical processes. Many of these processes are illustrated by a consideration of the melting that takes place at the roof, and we focus attention on this situation in this section. We shall assume that the container is sufficiently tall that any relevant positive Rayleigh numbers will be sufficiently large that the associated convection is turbulent. The calculation of the rate of melting of the roof is the main quantitative aim of the analysis.

Three separate cases can be considered. The melt may be heavier than the fluid and initially sink through it. The intense motion in the fluid then mixes the falling melt in with it. Alternatively, the melt may be less dense than the fluid and form a separate layer between the roof and the fluid. This melt layer can itself be in quite vigorous convective motion. An intermediate case is also possible, wherein the melt is initially denser than the fluid, and thus sinks. As the temperature of the falling fluid increases and its density thereby decreases, it becomes less dense than the surrounding fluid and rises. I have carried out experimental simulations of each of these three cases and will describe them briefly here. The experiments employed a roof of either wax or ice which was melted by the hot aqueous salt solution beneath it. The second case, that of a light melt, has important geological applications. It describes the melting of the continental crust by the emplacement of a hot, relatively dense input of fluid basaltic rock. Both the basaltic layer and the resultant granitic melt layer crystallize and increase their viscosities as they cool. These effects can be incorporated into the analysis and the rate of melting and the temperatures of the two layers can be calculated as functions of time.

4.1 Heavy melt

Consider a layer of hot fluid with initial temperature T_0 and vertical thickness H and assume that the solid roof above it is of very large thickness. For the case in which the melt mixes intimately with the fluid, we seek the subsequent mean temperature $T(t)$ of the fluid/melt mixture and its thickness $H + a(t)$ as functions of time. Figure 3 presents a sketch of the geometrical set-up. We assume that the problem can be considered as one-dimensional and that the thermal Rayleigh number of the fluid is sufficiently large that it convects turbulently. Then the heat flux F from the layer to the solid can be evaluated from the four-thirds relationship (Turner 1973)

$$F = \rho c J (T - T_m)^{\frac{4}{3}}, \tag{4.1}$$

where ρ is the fluid density, c the specific heat, T_m the melting temperature of the roof, and thus the temperature at the solid/melt interface, and

$$J = \gamma \left(\alpha g \kappa^2 / \nu\right)^{\frac{1}{3}}, \tag{4.2}$$

where α is the coefficient of thermal expansion of the fluid, g the acceleration due to gravity, κ the thermal diffusivity, ν the kinematic viscosity, and γ a dimensionless

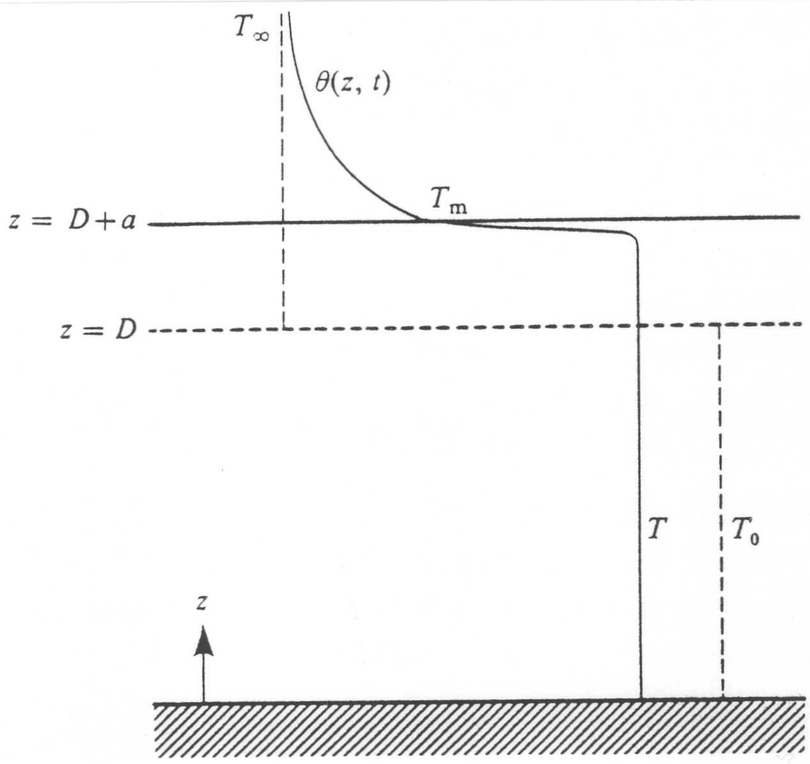

FIGURE 3. A sketch of the initial and subsequent geometry and temperature profile when a solid roof melts so that the melt density exceeds the fluid density.

constant approximately equal to 0.1 (Denton and Wood, 1979). We have neglected both the influence of the mass flux from the roof, on the assumption that the motion is dominated by the thermal transfers, and the variation in all physical properties as melt is incorporated into the fluid.

The flux F melts the solid at a rate that can be calculated by solving the thermal diffusion equation in the solid and applying conservation of heat at the melt interface. With the assumption that the melt rate is quasi-steady, the diffusion equation can be solved analytically (Holman 1976; Huppert 1986) to yield

$$\dot{a} = F/H_s = A(T - T_m)^{\frac{4}{3}}, \qquad (4.3\text{ a, b})$$

where

$$A = \rho c J/H_s, \quad H_s = \rho_s\left[c_s(T_m - T_\infty) + L\right], \qquad (4.4\text{ a, b})$$

ρ_s is the density of the solid, c_s its specific heat, L its latent heat on melting and T_∞ the temperature in the solid far from the solid/melt interface. Equation (4.3 a) states that the rate of melting is given by the ratio of the heat input F to the heat needed to bring the solid up to its melting temperature T_m from T_∞ and then to melt it. While (4.3) is only strictly valid if $\dot{a}$ is constant, results obtained using it in the following calculations were negligibly different from those obtained by solving the full diffusion equation. The use of (4.3), however, has the large advantage that analytical representations of the results can be obtained.

With the assumption that there is no heat loss from the base of the fluid, conservation of heat indicates that

$$\dot{a}(T-T_m)+(H+a)\dot{T}+J(T-T_m)^{\frac{4}{3}}=0. \tag{4.5}$$

The first term of (4.5) represents the rate at which heat is used to raise the melt from the melt temperature T_m to that of the melt/fluid mixture T. The second term represents the rate at which the heat content of the melt/fluid mixture changes. The third term represents the heat flux from the mixture to the overlying roof.

Equations (4.3 b) and (4.5), together with the initial conditions

$$T=T_0, \quad a=0 \quad (t=0), \tag{4.6 a, b}$$

specify the problem. The introduction of the non-dimensional variables

$$\theta=\frac{T-T_m}{T_0-T_m}, \quad \eta=\frac{C}{H}a, \quad \tau=ACH^{-1}(T_0-T_m)^{\frac{4}{3}}t, \tag{4.7 a, b, c}$$

where

$$C=\frac{H_s}{\rho c(T_0-T_m)}, \tag{4.8}$$

into (4.3)–(4.6) leads to

$$(C+\eta)\frac{d\theta}{d\tau}+\theta^{\frac{7}{3}}+C\,\theta^{\frac{4}{3}}=0, \tag{4.9}$$

$$\frac{d\eta}{d\tau}=\theta^{\frac{4}{3}}, \tag{4.10}$$

$$\theta=1, \quad \eta=0 \quad (t=0). \tag{4.11}$$

The one non-dimensional parameter, C involved in the solution represents the ratio of two quantities. The first is the heat required to bring solid of thickness H to its melting temperature and then melt it. The second is the initial heat content of the hot fluid with respect to the melting temperature of the roof.

Substituting (4.10) into (4.9), we obtain

$$\frac{d}{d\tau}[(C+\eta)\theta]+C\frac{d\eta}{d\tau}=0, \tag{4.12}$$

the first integral of which, upon using (4.11), becomes

$$(C+\eta)\theta+C\eta=C. \tag{4.13}$$

Substituting (4.13) into (4.10), we obtain

$$\frac{d\eta}{d\tau}=C^{\frac{4}{3}}(1-\eta)^{\frac{4}{3}}(C+\eta)^{-\frac{4}{3}}. \tag{4.14}$$

From (4.13) and (4.14) it can be seen that

$$\theta\rightarrow 0, \quad \eta\rightarrow 1 \quad (\tau\rightarrow\infty) \tag{4.15}$$

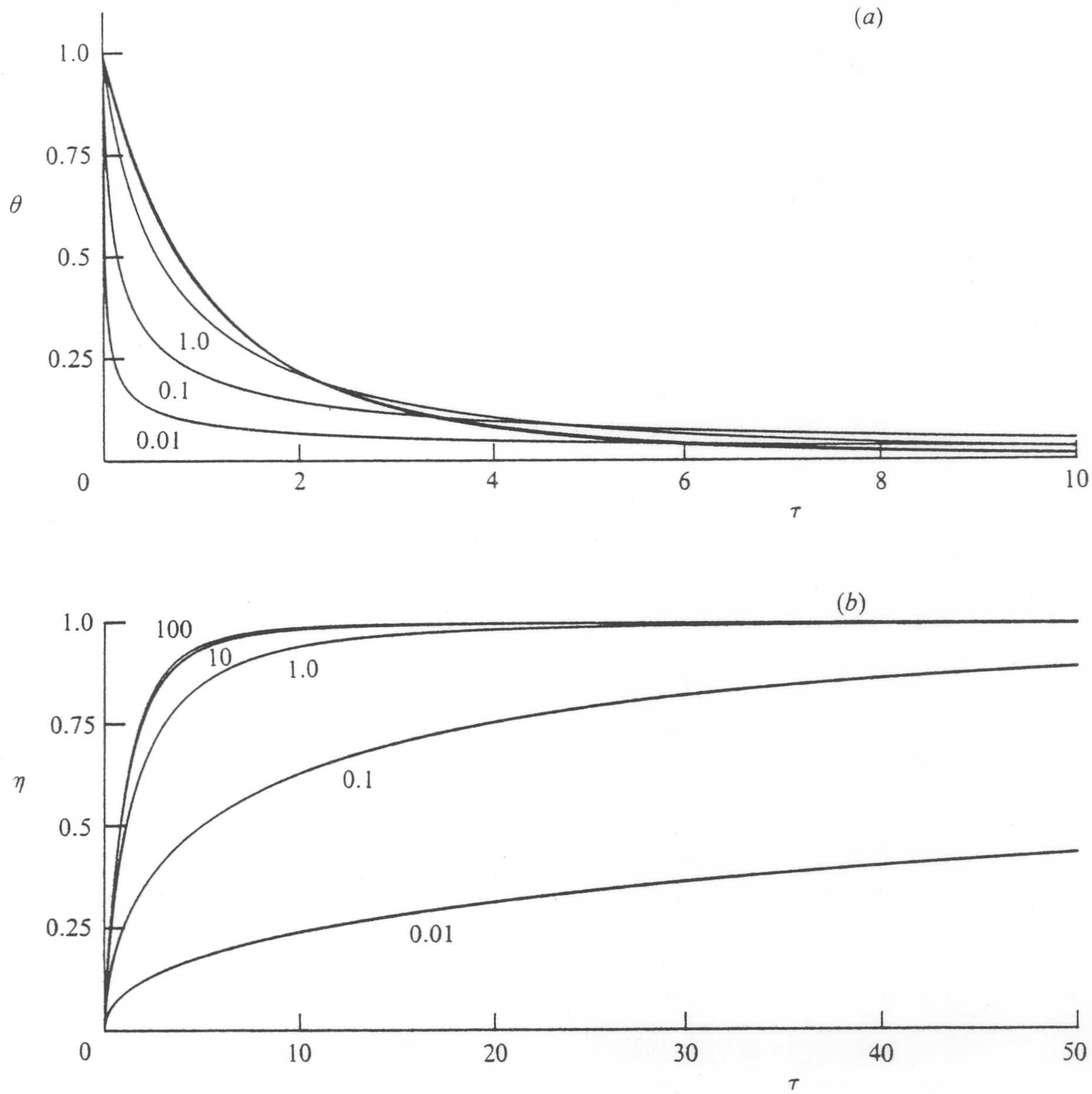

FIGURE 4. (a) The non-dimensional temperature $\theta = (T - T_m)/(T_0 - T_m)$ and (b) melt thickness $\eta = (C/H)a$ as a function of the non-dimensional time $\tau = (AC/H)(T_0 - T_m)^{3/4}t$ for $C = 0.01,\ 0.1,\ 1.0,\ 10$ and 100.

for all (positive) C. The interpretation of (4.15) is that eventually the temperature in the melt/fluid mixture approaches T_m, at which time all the initial excess heat content of the fluid has been used in producing the melt. In dimensional terms, a layer of thickness $C^{-1}H$ has been melted from the roof and incorporated into the fluid — a result that can be obtained directly by consideration of conservation of heat. Numerical solutions of (4.11), (4.13) and (4.14) are presented in figure 4.

In order to test the theoretical predictions presented in figure 4, we carried out a series of experiments with a wax roof. In all cases, the roof was made in a wooden mould of dimensions $20 \times 20 \times 15$ cm. When ready, the roof was lowered 10 cm into a thermally insulated Perspex container, to leave a $20 \times 20 \times 40$ cm high space beneath it. The wax used was polyethylene glycol, PEG 1000, which is miscible with water and has a melting temperature range of 37-40°C. Calibrations that we conducted indicate that the density of the solid wax is 1.21 g cm^{-3} at room temperature and decreases in the molten state fairly linearly between 1.109 g cm^{-3} at 40°C to 1.097 g cm^{-3} at 55°C. All the solutions were introduced at approximately 70°C, with small amounts of solution added periodically at the base of the container so as to compensate for the decrease in volume as the fluid cooled. The fluid level was in this way maintained in contact with the base of the wax. The experiments were monitored until most of the wax had melted, by which time the temperature in the solution had fallen by some 10–15°C.

As the wax melted, the retreating base of the solid block became slightly uneven. Nevertheless, a measurement of the 'mean' height of the base could be taken fairly reliably by eye. The results from two experiments are depicted in figure 5 (a, b), which also graphs the corresponding theoretical result obtained from the numerical integration of (4.11), (4.13) and (4.14). The agreement is seen to be good.

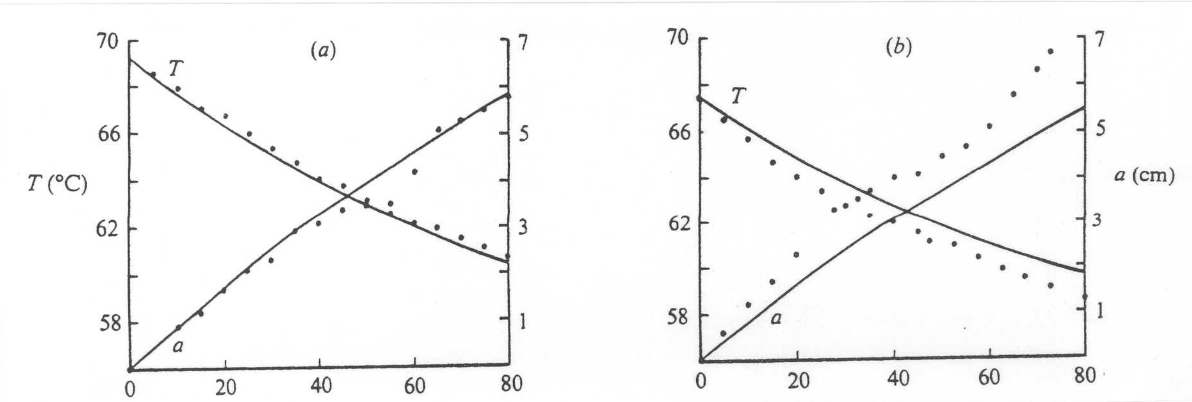

FIGURE 5. Experimental (.) and theoretical (—) temperatures and melt thickness as functions of time in minutes for two experiments with a wax roof.

4.2 Light melt

If the melt density is less than the fluid density, it will form a separate layer between the roof and the hot fluid. Initially the Rayleigh number in this intermediate layer will be less than the critical value necessary for convection and the melt will transfer heat only by conduction, even though the fluid beneath it is in vigorous turbulent motion. There may be a weak transfer of material across the interface, by the entrainment mechanisms reviewed by Turner (1973, Ch. 9), but it was not detected in any of our experiments. After a time the Rayleigh number of this intermediate layer becomes sufficiently high that it begins to convect and some time later it will convect turbulently.

An analysis of this whole situation can be quite directly carried out, which is in general not dissimilar to that described in section 4.1, the details of which are elaborated in Huppert and Sparks (1988a).

We conducted three experiments in which the melt was always less dense than the underlying fluid. The effects observed in all these experiments was satisfactorily consistent with the processes and descriptions outlined.

4.4 Melt of intermediate density

In two of the experiments, the wax at its melting temperature was slightly more dense than the hot fluid and initially sank. As it did so its temperature increased and its density correspondingly decreased. This decrease could be sufficient to make parts of the molten wax less dense than the surrounding fluid, causing it to rise. The form of motion was similar to the upward-moving blobs observed during various investigations in which light fluid has been released beneath heavier fluid (see, for example, Whitehead and Luther 1975; Marsh 1979; Kerr and Lister 1988).

5. HOT FLUID FLOWS OVER ERODIBLE BEDS

There are many situations, both in industry and in nature, where a rapidly flowing hot liquid spills out over a solid base whose melting temperature is less than the temperature of the liquid. The solid can then possibly melt and become incorporated into the liquid. Alternatively, the liquid may be so effectively cooled by the underlying solid that it solidifies first. It is interesting to calculate the conditions under which each alternative occurs, or indeed if one possibility occurs first and is then followed by the other.

Practical examples of this situation include the following. First, the dispersion of hot water over solid ice in order to melt and clear the ice, and second, the pouring of a hot chocolate sauce over cold ice cream. On a more voluminous scale, many cubic kilometres of liquid rock at a temperature of around 1600° C were erupted in the Archean age (4.5 to 3.0×10^9 years ago) on the then sea floor which consisted of rocks having a melting temperature of around 750° C. The molten lava flow become contaminated with the base rock to form many of the deposits of nickel found on the earth, as first postulated by Huppert *et al.* (1984). The important part of the process was that nickel brought up from within the Earth in very small concentrations by the eruption combined with sulphur supplied by the melted base rocks to form nickel sulphide. The nickel sulphide droplets formed are immiscible in the lava and significantly more dense than it. They would thus have precipitated at the base of the flow, to be subsequently mined, in particular at Kambalda in Western Australia, many years later.

To return to the initial value problem, however, an interesting paradigm calculation can be made to show that in general the flowing liquid *must* first solidify, though it may then be rapidly re-melted. Consider the liquid, which is assumed to be flowing turbulently, to be at uniform temperature T_∞ and the solid, of semi-infinite extent, to be initially at uniform temperature T_0. For simplicity we assume that the melting temperature of the solid, T_s, equals the solidification temperature of the liquid, T_f. The finite heat flux into the interface between the liquid and solid, H, occurs simultaneously with a large (initially infinitely large) conductive heat flux, F, in the solid away from the interface. The fluxes H and F can only be balanced by the latent-heat release provided by ***solidification*** of the base of the turbulent flow. We thus argue that solidification of the fluid must be the first response (initially at an infinite rate). As time proceeds, however, F decreases monotonically as the conductive temperature profile relaxes. There will thus come a time when H first exceeds F and subsequently melting at the interface occurs. The previously solidified layer will melt first, followed by the original solid.

Quantitative analysis, which is described in detail in Huppert (1989), indicates that the maximum thickness of the resultant solid scales with the quantity

$$h_m = k^2(T_f - T_0)^2/\rho\kappa HL$$

and its evolution takes place on a timescale

$$t_s = k^2(T_f - T_0)^2/\kappa H^2 \ ,$$

where k is the thermal conductivity, κ the thermal diffusivity, ρ the density and L the latent heat, with all these material properties assumed to be equal for liquid and solid. Subsequently, for $t \gg t_s$, the melting interface propagates at a constant speed given by

$$V = \frac{H}{\rho c[T_m - T_\infty + Lc^{-1}]} \tag{5.1}$$

(c.f. equations 4.3 and 4.4).

The addition of an equation describing the heat budget of the flow to that describing the velocity of melting permits one to solve for the temperature of the liquid, and also that in the solid, as a function of the downstream flow direction. In addition, the relative contamination, or volume of melted ground added to the current per unit volume of initial material in the current, can be calculated. For physical parameters relevant to Archean geology, such a theoretical calculation predicts that the relative contamination varies from 0 at source (where no material has yet had a chance to be added) to a typical figure of approximately 20%. This is in good agreement with field observations.

6. CONCLUSIONS

This paper has but touched on the many situations in geology and geophysics for which heat and mass transfer problems play a fundamental role. No doubt many more rich and exciting topics remain to be explored. The applied mathematician, engineer or fluid dynamiscist who is willing to learn a little background information about the Earth is in an ideal position to make dominant contributions in this blossoming field.

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