reflectors; had they been absorbers of sound, they would have left a 'hole' in the noise field and the difference signal would have been zero or negative. A negative signal can form the basis of an image, a 'silhouette', the important criterion being that there should be an acoustic contrast between the object and its background.

Although this first acoustic daylight experiment in the ocean was successful, the observed spectrum is a far cry from a pictorial image on a television monitor. In effect, we have generated just one pixel of an image, corresponding to the single beam of the parabolic reflector. More pixels require more beams. These could be produced with a volumetric phased array, although other possibilities involving reflectors, either parabolic or spherical, and refractive acoustic lenses are also being considered. □

Received 6 November 1991; accepted 11 February 1992.

1. Urick, R. J. *Principles of Underwater Sound* (McGraw-Hill, New York, 1975).
2. Kerman, B. *Sea Surface Sound* (Kluwer, Dordrecht, 1988).
3. Melville, W. K., Loewen, M. R., Felizardo, F. C., Jessup, A. T. & Buckingham, M. J. *Nature* **336**, 54–56 (1988).
4. Mellen, R. H. *J. acoust. Soc. Am.* **24**, 478–480 (1952).
5. Buckingham, M. J. & Jones, S. A. S. *J. acoust. Soc. Am.* **81**, 938–946 (1987).
6. Glegg, S. A. L. & Buckingham, M. J. *Florida Atlantic Univ. Rep.* (July 1990).

ACKNOWLEDGEMENTS. We thank E. Slater and L. Green for the loan of the parabolic reflector and G. Deane for guidance on design of circuit elements. This work was supported by the Office of Naval Research.

# Analytical model for solidification of the Earth's core

**Bruce A. Buffett, Herbert E. Huppert, John R. Lister & Andrew W. Woods**

Institute of Theoretical Geophysics, Department of Applied Mathematics and Theoretical Physics and Department of Earth Sciences, University of Cambridge, Cambridge CB3 9EW, UK

**THE Earth's solid inner core is generally thought to have formed by gradual solidification of the liquid core as the Earth cooled[1–3]. To elucidate the relative importance of the various physical effects on the thermal evolution of the core, we have developed an analytical model based on global heat conservation, which describes the cooling of the vigorously convecting, fluid outer core and the concomitant growth of the inner core. We obtain a simple form for the evolution of the inner-core radius which allows the consequences of changes to the model's input parameters to be readily assessed. For most of this evolution, inner-core growth is controlled primarily by the heat capacity of the outer core and the history of the heat flux into the base of the mantle. Heat sources associated with solidification of the inner core, including the release of latent heat and gravitational energy, have a secondary role but become more important towards the end of solidification. Using current seismic estimates of compositional changes at the surface of the inner core, we conclude that the compositional and thermal buoyancy fluxes in the outer core are comparable.**

The energetics of the cooling core have been studied previously[4–7] to estimate the energy available to power the geodynamo through the release of latent heat on solidification[8] and through the associated generation of compositional buoyancy[9] by the exclusion of light elements from the iron-rich inner core. Other studies[10–12] have attempted to construct thermal histories of the Earth's core using parameterized models of convection. The latter have all involved extensive numerical calculations, in contrast to the analytical model developed here.

In our model we assume that compositional and thermal buoyancy forces drive vigorous convection in the liquid outer core, and that the liquid core is well mixed with an adiabatic temperature profile. The potential temperature in the liquid core, which is defined as the temperature after subtracting the adiabatic variation, is therefore spatially uniform and slowly decreases with time (Fig. 1). We also assume that the surface of the inner core is in thermodynamic equilibrium with the surrounding liquid. With these two assumptions, the temperature through the outer core is uniquely determined by the solidification temperature, $T_s(p)$, which we treat as a function of pressure $p$ only and hence implicitly of depth. The solidification temperature may also be a function of the slowly varying composition. At present, the details of the phase diagram for the iron alloy constituting the core are poorly constrained, but simple theoretical models suggest that the effect of compositional variations on $T_s$ is small[7].

The heat extracted from the cooling outer core is transported through the overlying mantle to the surface of the Earth by thermal convection. Convective motions in the mantle[13] are characterized by effective viscosities of $10^{21}$–$10^{23}$ Pa s (refs 14–16). The cooling timescale associated with mantle convection is therefore much greater than that characteristic of convection in the outer core, where the viscosity is less than $5\times10^3$ Pa s (ref. 17). Thus the relatively sluggish and more massive mantle controls the cooling of the core by regulating the heat flux $f_m(t)$ across the core–mantle boundary. The magnitude and time dependence of $f_m(t)$ will depend on the details of mantle convection, which include the distribution of radioactive isotopes and any compositional or rheological layering in the mantle. As many such details are unknown, we treat $f_m(t)$ as a prescribed parameter and focus on the evolution of the core. Different models of mantle convection will give rise to different values of $f_m(t)$, and these may be readily incorporated into our model.

The model is based on global heat conservation, which equates the heat lost from the core liquid plus the heat produced by growth of the inner core to the net heat flux from the outer core. The main sources of heat associated with the growth of the inner core are the latent heat released by solidification and the loss of gravitational energy due to the solidification of an iron-rich inner core. This gravitational energy is converted to heat by buoyancy-driven motions in the core. Further gravitational energy is released by adiabatic compression and thermal contraction of the Earth, but detailed numerical calculations (B.A.B., H.E.H., J.R.L. and A.W.W., manuscript in preparation) show that these additional effects are less important, and they are not considered further here. Radioactive heat sources in the core[18] could be explicitly included in the model, but such heat

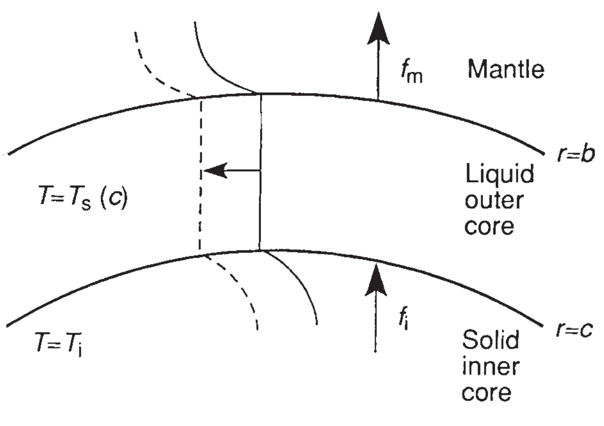


FIG. 1 Schematic profile of the potential temperature (the temperature after subtracting the adiabatic variation) in the slowly cooling core of the Earth. The solid line represents the temperature in the core and lower mantle at some instant. The dashed line represents the subsequent evolution of temperature as heat is continuously extracted from the core. Heat fluxes $f_m$ and $f_i$ pertain to the core–mantle and inner-core boundaries respectively. The solidification temperature $T_s(c)$ is defined at the inner core boundary, and $T_i(r, t)$ is the temperature within the inner core.

sources can be absorbed into $f_m$. Thus, in spherical geometry, we express the global heat balance as

$$\frac{4\pi}{3}(b^3-c^3)C_p\frac{\mathrm{d}T_s}{\mathrm{d}t}-4\pi c^2(L+B)\frac{\mathrm{d}c}{\mathrm{d}t} = 4\pi c^2 f_i(t) - 4\pi b^2 f_m(t) \quad (1)$$

where $C_p$ and $L$ are the specific heat and latent heat per unit volume, and $c$ and $b$ are the inner and outer radii of the liquid core. The release of gravitational energy per unit volume $B$ is due to the compositional buoyancy flux at the inner-core boundary. The change in the gravitational energy that results from the rearrangement of mass within the core can be approximated by[2]

$$B = \Delta\rho g(b) b[\tfrac{3}{10} - \tfrac{1}{2}(c/b)^2] \quad (2)$$

where $\Delta\rho$ is the compositional density jump across the inner-core boundary and $g$ is the local acceleration due to gravity. The conductive heat flux $f_i$ from the inner core is obtained from

$$\mathbf{f}_i(t) = -k\nabla T_i(c, t) \quad (3)$$

where $T_i(r, t)$ is the temperature within the solid inner core and $k$ is the thermal conductivity. Time $t$ is measured from the instant at which the temperature of the core first falls below the solidification temperature at the centre of the Earth and the inner core begins to grow.

We circumvent the need for an explicit solution for the temperature in the interior of the inner core by considering two bounding cases for $f_i$, and hence for $T_i$. These two endmembers correspond physically to the limits of: (1) a perfectly insulating inner core, $k\to 0$; and (2) a perfectly conducting inner core, $k\to\infty$. In the former case, no heat is conducted from the inner core and $f_i = 0$. In the latter case, heat is readily conducted from the inner core, thus maintaining an isothermal temperature field within the solid given by the solidification temperature $T_s$ at the interface. Consideration of the heat content of the inner core then shows that $f_i \to -\frac{1}{3}cC_p(\mathrm{d}T_s/\mathrm{d}t)$ as $k\to\infty$.

The equation governing the radius $c(t)$ of the inner core is obtained by making the radial dependence of $T_s(p)$ explicit in equation (1). The relationship between $p$ and $r$ is determined by the hydrostatic equation $\partial p/\partial r = -\rho g$ which dominates the radial momentum balance of the Earth, where $\rho$ is the mass density and $g$ can be obtained from Newton's law of gravitation. In our calculations we neglect the known radial and temporal variations in density because their effects are small compared with both the leading order radial variation and the uncertainties in $T_s(p)$. Over a limited range of pressure, $T_s(p(r))$ can be approximated using a Taylor series expansion about $p(0)$. Hence to first order, the radial dependence of $T_s$ is

$$T_s(r) = T_s(0) - \frac{2\pi}{3}G\rho^2 r^2\frac{\partial T_s}{\partial p} \quad (4)$$

where $G$ is the gravitational constant.

Equation (4) relates the temperature of the outer core to the radius of the inner core. Substituting (4) into (1), we obtain equations for the growth of the inner core which can be integrated directly in the two limiting cases, $k\to 0$ and $k\to\infty$. For these two cases, $c(t)$ is given by

$$\left(\frac{c}{b}\right)^2\left[1-\frac{2}{5}\left(\frac{c}{b}\right)^3\right]+\mathcal{L}\left(\frac{c}{b}\right)^3+\mathcal{B}\left(\frac{c}{b}\right)^3\left[1-\left(\frac{c}{b}\right)^2\right] = \frac{1}{\mathcal{M}}\int_0^t f_m(\tau)\,\mathrm{d}\tau \quad (5)$$

and

$$\left(\frac{c}{b}\right)^2+\mathcal{L}\left(\frac{c}{b}\right)^3+\mathcal{B}\left(\frac{c}{b}\right)^3\left[1-\left(\frac{c}{b}\right)^2\right] = \frac{1}{\mathcal{M}}\int_0^t f_m(\tau)\,\mathrm{d}\tau \quad (6)$$

respectively, where

$$\mathcal{M} = \frac{2\pi}{9}b^3C_pG\rho^2\frac{\partial T_s}{\partial p},\quad \mathcal{L} = \frac{1}{3}\frac{Lb}{\mathcal{M}},\quad \mathcal{B} = \frac{1}{10}\frac{\Delta\rho g(b)b^2}{\mathcal{M}} \quad (7)$$

The parameter $\mathcal{M}$ is a measure of the heat that must be extracted to cool the entire liquid core to its solidification temperature. $\mathcal{L}$ and $\mathcal{B}$ represent the importance of the latent heat and the gravitational energy loss, respectively, relative to the heat due to cooling.

One of the most important features of our analytical model is the insight it offers into the processes that dominate the growth of the solid inner core. We can deduce immediately that differences between the two extreme models of a perfectly insulating and perfectly conducting inner core are negligible when $(c/b)^3$ is small. This condition is satisfied over most of the evolution of the core, and so the rate at which heat is lost from the inner core has an insignificant effect on the cooling of the core as a whole. Our analysis also indicates that the effects of latent heat release and loss of gravitational energy are a factor $c/b$ smaller than the heat extracted by cooling the core. For the parameter values given in Table 1, we find that $c/b = 0.35$, $\mathcal{L} = 0.5$ and $\mathcal{B} = 0.6$. This shows that the growth of the inner core is currently controlled by a balance between the heat flux into the base of the mantle and the decrease in the heat content of the cooling outer core. With this approximation, we obtain the following

TABLE 1 Physical properties of the Earth's core

| Parameter | | |
|---|---|---|
| Thermal expansion coefficient[23] | $\alpha$ | $7\times10^{-6}$ °C$^{-1}$ |
| Latent heat[24] | $L$ | $6.0\times10^{9}$ J m$^{-3}$ |
| Specific heat capacity[23] | $C_p$ | $6.7\times10^{6}$ J °C$^{-1}$ m$^{-3}$ |
| Solidification profile[24] | $\partial T_s/\partial p$ | $7\times10^{-9}$ °C Pa$^{-1}$ |
| Mean density[22] | $\rho$ | $1.2\times10^{4}$ kg m$^{-3}$ |
| Density jump at inner-core boundary[22] | $\Delta\rho$ | 600 kg m$^{-3}$ |
| Outer-core radius[22] | $b$ | 3,480 km |
| Current inner-core radius[22] | $c$ | 1,221 km |
| Gravitational constant | $G$ | $6.67\times10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ |
| Current radius ratio | $c/b$ | 0.35 |
| Model constant | $\mathcal{M}$ | $1.3\times10^{16}$ J m$^{-2}$ |
| Latent heat parameter | $\mathcal{L}$ | 0.5 |
| Gravitational energy parameter | $\mathcal{B}$ | 0.6 |

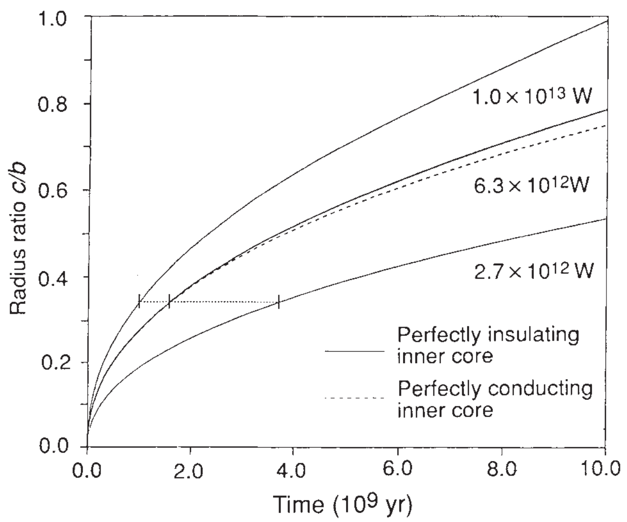


FIG. 2 Growth of the inner core measured from the time at which solidification begins. Three possible solutions are presented using a range of estimates for the net heat flux $4\pi b^2 f_m(t)$ across the core–mantle boundary (see text). The horizontal dotted line indicates the current radius of the inner core. Before $t = 0$, the core of the Earth was entirely liquid.

simple expression for the radius of the inner core

$$c(t)=b\left[\int_0^t f_m(\tau)\,d\tau/\mathcal{M}\right]^{1/2} \tag{8}$$

Specific predictions of $c(t)$ from our model require an estimate of $f_m(t)$. A plausible upper bound on $f_m$ can be inferred from the heat flux measured at the Earth's surface. Given chondritic abundances of radioactive isotopes[19] in the upper mantle, it is unlikely that the total heat flux across the core-mantle boundary exceeds $10^{13}$ W. It is also unlikely, although not impossible, that the total heat flux is less than that conducted up the adiabat, which is estimated[12] to be $2.7\times10^{12}$ W. As an illustrative example, we consider these two bounds on $f_m$, along with the midpoint value $6.3\times10^{12}$ W as a typical estimate. Using the parameter values in Table 1, we obtain the growth histories shown in Fig. 2. For the typical value of $f_m$, we see that the inner core grows to its present radius in $1.6\times10^9$ years, a value similar to those obtained previously by Stevenson *et al.*[10]. The values calculated from the bounds on $f_m$ are $1.0\times10^9$ and $3.6\times10^9$ years. These ages span a wide interval, but are roughly consistent with palaeomagnetic data[20], which are sometimes used to constrain the age of the inner core under the assumption that the onset of solidification is associated with a rapid increase in the strength of the magnetic field.

The thermal buoyancy flux associated with the cooling of the liquid core depends on the amount by which $f_m$ exceeds the heat flux $f_a$ conducted down the adiabatic gradient. For our typical value of $f_m$, the thermal buoyancy flux caused by the downwelling of cold dense fluid at the core-mantle boundary dominates that associated with latent heat release at the inner-core boundary. But an additional buoyancy flux of compositional origin is produced at the inner-core boundary by the release of light, residual fluid on solidification. The relative importance of the thermal and compositional contributions can be assessed using our model once $\Delta\rho$ and $f_m$ are known. Seismic estimates of the relative density jump $\Delta\rho/\rho$ vary from 2 to 12% (ref. 21), with typical values of 5% (ref. 22). The actual compositional density jump, however, may be less than these seismic estimates if there is any volume change on solidification. In terms of $\Delta\rho$, the compositional buoyancy flux is $4\pi\Delta\rho g(c)c^2(dc/dt)$. For comparison, the thermal buoyancy flux is given by $4\pi\alpha\rho g(b)b^2(f_m-f_a)$, where $\alpha$ is the coefficient of thermal expansion. From the range of possible values of $f_m$, we estimate that these two fluxes are roughly comparable if the seismic estimates of the density jump across the inner core boundary represent a compositional change. □

Received 3 January; accepted 20 February 1992.

1. Jacobs, J. A. *Nature* **172**, 297–298 (1953).
2. Loper, D. E. *Adv. Geophys.* **26**, 1–34 (1984).
3. Jacobs, J. A. *The Earth's Core* (Academic, London, 1987).
4. Gubbins, D. *J. Geophys.* **43**, 453–464 (1977).
5. Loper, D. E. *Geophys. J. R. astr. Soc.* **54**, 389–404 (1978).
6. Gubbins, D., Masters, T. G. & Jacobs, J. A. *Geophys. J. R. astr. Soc.* **59**, 57–99 (1979).
7. Loper, D. E. *J. Geomag. Geoelectr.* **43**, 79–91 (1991).
8. Verhoogen, J. *Geophys. J. R. astr. Soc.* **4**, 276–281 (1961).
9. Braginsky, S. I. *Dokl. Akad. Nauk. SSSR* **149**, 8–10 (1963).
10. Stevenson, D., Spohn, T. & Schubert, G. *Icarus* **54**, 466–489 (1983).
11. Mollett, S. *Geophys. J. R. astr. Soc.* **76**, 653–666 (1984).
12. Stacey, F. D. & Loper, D. E. *Phys. Earth planet. Inter.* **36**, 99–115 (1984).
13. Ashby, M. F. & Verrall, R. A. *Phil. Trans. R. Soc.* A**288**, 59–87 (1977).
14. O'Connell, R. J. *Tectonophysics* **38**, 119–136 (1977).
15. Wu, P. & Peltier, W. R. *Geophys. J. R. astr. Soc.* **70**, 435–486 (1982).
16. Hager, B. H. *J. geophys. Res.* **89**, 6003–6015 (1984).
17. Gwinn C. R., Herring, T. A. & Shapiro, I. I. *J. geophys. Res.* **91**, 4755–4765 (1986).
18. Lewis, J. S. *Earth planet. Sci. Lett.* **11**, 130–134 (1971).
19. Stacey, F. D. *Physics of the Earth*, 2nd edn (Wiley, New York, 1977).
20. Hale, C. J. *Nature* **329**, 233–237 (1987).
21. Masters, T. G. & Shearer, P. M. *J. geophys. Res.* **95**, 691–721 (1990).
22. Dziewonski, A. M. & Anderson, D. L. *Phys. Earth planet. Int.* **25**, 297–356 (1981).
23. Stacey, F. D. *Phys. Earth planet. Int.* **15**, 341–348 (1977).
24. Verhoogen, J. *Energetics of the Earth* (National Academy of Science, Washington, DC, 1980).

ACKNOWLEDGEMENTS. We thank B. A. Bolt, D. O. Gough, D. Gubbins, J. A. Jacobs, R. Jeanloz, D. E. Loper, D. P. McKenzie, H. K. Moffatt, W. H. Munk and D. J. Stevenson for helpful comments. This work was supported in part by the British Petroleum Venture Research Unit, and by a Visiting Professorship to H.E.H. in the School of Mathematics, University of New South Wales, Australia.

# Parent–offspring conflict and the recruitment of helpers among bee-eaters

**Stephen T. Emlen & Peter H. Wrege**

Section of Neurobiology and Behavior, Cornell University, Ithaca, New York 14853, USA

**GENETIC conflicts of interest are to be expected between individuals in any non-clonal society[1–5]. One well studied form of conflict is that between parents and their offspring over the amount of parental care provided to the offspring[3]. A very different manifestation of parent–offspring conflict may occur in certain cooperatively breeding species in which parents (breeders) are assisted in the rearing of young by their grown offspring (helpers)[6–8]. If helpers have a sufficiently large effect on reproductive success, breeders will enhance their own inclusive fitness more by retaining their offspring as helpers than by allowing them to reproduce on their own[4,5,9]. We report here that older male white-fronted bee-eaters (typically fathers) actively disrupt the breeding attempts of their sons, and that such harassment frequently leads to the sons joining as helpers at the nest of the harassing father. Calculation of fitness costs and benefits to the various participants helps to clarify both why parents engage in such 'recruitment' behaviour and why sons frequently do not resist.**

White-fronted bee-eaters (*Merops bullockoides*) are common birds of the savannas of East and Central Africa. They breed both gregariously (in colonies averaging 200 individuals) and cooperatively (with half of all nesting attempts including non-breeding adult helpers)[10]. Young remain with their parental group until they pair at one to two years of age, when females (but not males) disperse to join the group of their mate[11]. The resulting social organization is one of patrilocal extended family groups (clans). Each clan occupies a stable feeding territory, but commutes to roost and nest at the colony site[11,12].

During the breeding season, several pairs in each extended family may initiate breeding. Some individuals suffer harassment from other members of their clan and many initial reproductive attempts are abandoned. We examined the frequency, pattern and consequences of these harassments to test the hypothesis that such interference constitutes a form of parental manipulation to recruit grown offspring into becoming helpers.

We collected observations ad lib during 5 and 1 breeding seasons, respectively, from two individually marked and genealogically known populations of white-fronted bee-eaters at Lake Nakuru National Park, Kenya. We defined harassment as involving one or more of the following. (1) Prolonged aggressive chasing of a resident bird on the latter's territory. (2) Repeated interference by males in the courtship feeding of another pair. Such behaviour took the form of either physically preventing the transfer of food to the female, or of begging (but not taking food) from the allofeeding male. Similar begging was also directed towards single birds. Because begging females accepted allofed insects, alternative nutritional explanations for their behaviour could not be discounted. Consequently, cases based only on begging evidence by females were excluded. (3) Blocking a pre-breeding pair from access to its nest chamber. In this behaviour the harassing individual would position itself at a nest entrance and either chase or deny access to the occupants as they attempted to enter. (4) Repeated visits, typically accompanied by begging or other vocalizations, to the nest chamber of another pair during the period before egg laying or hatching at the nest of the harasser. (5) In some analyses we also included cases ($N=9$) in which harassment was implied but not directly observed: one member of an incubating pair abandoned its nest and began helping elsewhere while its mate continued to tend the clutch alone.