11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

SEDIMENTATION AND MIXING AT THE TOP OF A LAYER OF SUSPENDED PARTICLES

J. Stewart TURNER[1] and Herbert E. HUPPERT[2]

[1]Research School of Earth Sciences, Australian National University
GPO Box 4, Canberra, ACT 2601, AUSTRALIA

[2]Institute of Theoretical Geophysics, Dept of Applied Mathematics & Theoretical Physics
University of Cambridge, 20 Silver St, Cambridge CB3 9EW, UNITED KINGDOM

ABSTRACT

Even modest concentrations of solid particles can significantly affect the density of the fluid layers in which they are suspended, and hence the rate of mixing of the suspension with overlying fluid. Laboratory and theoretical results have been obtained for two cases that are important in a variety of natural and industrial applications. In the first experiment, we considered a suspension overlain by fluid of density intermediate between that of the bulk suspension and the interstitial fluid. Settling of the dense sediment leaves behind a thin boundary layer of lighter fluid which convects upwards, carrying some of the sediment with it. The interface at the top of the sedimenting layer remains sharp and descends at a constant rate. In a second set of experiments the interstitial fluid and the fluid above has the same density, with the lower, denser sediment layer being stirred by an oscillating grid. The long term steady state for a given size and concentration of particles depends on the stirring frequency and consequent intensity of turbulence. When the stirring rate is low all particles are precipitated on the floor, but over a limited frequency range a fraction of the particles is retained in a stable self-maintained suspension layer with a sharp density interface above it. As the stirring frequency is increased further all particles are held in suspension in a layer of fixed depth, while at still higher stirring rates the layer depth increases without limit and there is a corresponding decrease in particle concentration.

INTRODUCTION

The problem of mixing across a density interface has received a great deal of attention because of its importance in atmospheric, oceanic and industrial applications. Most of the laboratory and theoretical work has concentrated on fluid interfaces, where the density difference is produced by the variations either in temperature or salinity or both. Investigations have been carried out both for gravity currents, in which the turbulence producing the mixing is generated by the bottom stress, and for conditions of no mean flow, with the turbulence being generated by an oscillated stirring grid.

There are, however, many situations where suspended particles in a gas or liquid have a substantial effect on the bulk density, either alone or in combination with density differences in the fluid itself. One particular motivation for the present investigations is the understanding of sedimentation from rivers as they flow into the sea. Under some conditions a freshwater outflow, heavily laden with sand and silt, will initially underlie the seawater. The settling of the suspended load can then lead to convective mixing above the sediment layer. The laboratory experiments described below shed light on both the criterion for the continued suspension of particles in such a turbidity current, and the subsequent convective behaviour. In additon the results will be applicable to a wide variety of other sediment-laden flows including evolving crystal-rich layers in magma chambers, pyroclastic flows, avalanches and the deposition of industrial wastes.

CONVECTION DRIVEN BY DIFFERENTIAL SEDIMENTATION

Laboratory Observations

We consider first the one-dimensional problem with no mean flow, suggested by the example of a sediment-laden fresh-water layer under salt water. Specifically, the initial state used in our experiments is a suspension of small dense particles overlain by a clear sugar solution whose density ρ_U is greater than that of the interstitial fluid ρ_I, but less than that of the bulk suspension ρ_B. (Sugar was used instead of salt to avoid the coagulation of particles which was experienced in preliminary experiments using the latter solute.) This system was set up by adding the suspension below the sugar layer, using a pipe leading from a stirred reservoir to the bottom of the tank. The filling was completed in less than a minute, producing a sharp interface and negligible mixing with the upper layer.

The particles used were two grades of commercially available carborundum grinding powders of irregular shape, with effective median diameters of 8.7μm and 25.0μm and size distributions about these values which were determined using the sedigraph technique (Jones et al. 1988) to measure the settling velocities. The settling of particles away from the interface leaves behind buoyant interstitial fluid, which rises in thin streamers and sheets and drives vigorous convection in the upper layer, as shown in Fig.1. Some of the particles are carried across the interface with the light fluid, and mixed vigorously through the entire depth of the upper layer. There is no transport of fluid from the upper region into the lower, in which unimpeded settling onto the bottom of the tank continues throughout the experiment, at the same rate as it does without an overlying sugar layer. (See Fig.2.)

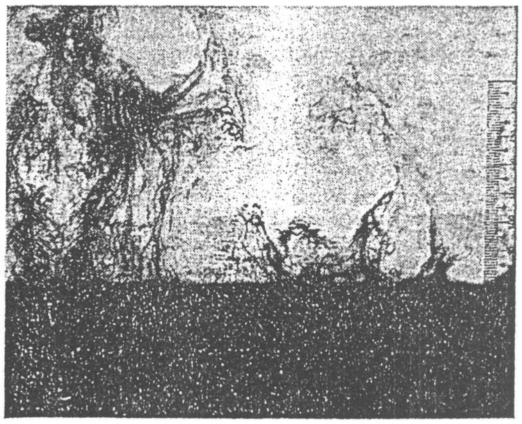

Fig. 1. Convection in an aqueous sugar solution, driven by sedimentation in an underlying suspension of polydisperse carborundum particles in pure water. The photograph was taken 45s after the start of the experiment.

The majority of the experiments were conducted in a perspex tank consisting of a lower region 19.5cm x 3cm in cross section and 20cm deep, joined at the top to a much larger tank, as described in detail by Huppert et al. (1991). The purpose of this larger region was to provide an upper fluid layer of such large volume that its properties (in particular the density ρ_U) did not change significantly during the course of an experiment. With this configuration, the velocity of the interface between the sediment layer and the convecting upper layer was found to descend at a constant velocity, which depends systematically on the densities of the two layers and the distributions of particle settling velocities, in a manner which can be understood in terms of the theory summarized below.

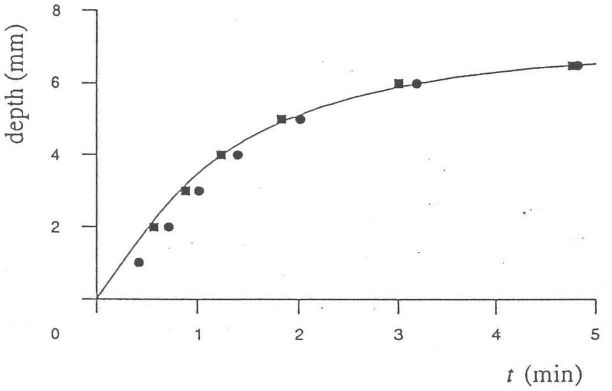

Fig. 2. The depth of sediment accumulated at the base of the tank in experiments with the same lower sediment layer ρ_B=1.076gcm^{-3} and a) no upper layer (circles) and b) an upper layer of sugar solution ρ_U = 1.030gcm^{-3} (squares). The evolution of the two is identical, after allowing for uncertainties in the starting times, and is well described by the accompanying theoretical curve.

Theoretical Description

The theory depends explicitly on the polydisperse nature of the particles. Consider first the simpler case where the fluid density is constant throughout, i.e. the interstitial and upper layer fluid have the same density $\rho_I = \rho_U$. Large particles can sediment away from the interface faster than small particles, so once sedimentation has started a density gradient begins to evolve at the top of the lower region. Below the fastest moving particle which started at the interface all the particle species are present in their original concentrations, but each successive layer above contains a more limited size range. For dilute suspensions each particle can be assumed to fall nearly independently of the others at the Stokes settling velocity. The positions of the boundaries of identified size ranges and the total volume fraction of particles (and hence the bulk density) in each layer can thus be calculated straightforwardly as a function of time; and corrections can be made for "hindered settling" in more concentrated suspensions. The interfaces between layers defined in this way fall at a constant velocity which is smaller for the successively higher layers.

Now consider the case in which $\rho_B > \rho_U > \rho_I$. The initial sedimentation will still yield a series of layers containing different distributions of particle sizes, but some of the uppermost layers will now be less dense than the overlying fluid. As the interfaces between the layers propagate downwards, the vertical scale of these buoyant layers will increase linearly with time. When a critical depth is reached, the buoyant layers will become unstable according to the Rayleigh-Taylor mechanism, detach and mix convectively with the overlying fluid. At the same time, a number of the (negatively buoyant) layers below the unstable region may also be entrained, by viscous or inertial coupling. Following the detachment of a certain number of layers, the same particle distribution and density structure will develop above the new sharp interface, until again the upper region becomes unstable. Since the detachment depends on the density structure, and this structure is the same from cycle to cycle, exactly the same number of layers will detach each time. It follows that the interface between the convecting and sedimenting regions will fall at a constant velocity V, as is observed.

We can place theoretical bounds on this constant fall velocity. Let us consider a dimensionless density ratio

$$R = (\rho_U-\rho_I)/(\rho_B-\rho_I) \quad . \qquad (1)$$

A lower limit on V, V_{min}, corresponds to the condition of no entrainment of negatively buoyant fluid, so that all fluid less dense than ρ_U, and no more, become detached and convect upwards. An upper bound, V_{max}, is given by the condition that the integrated, net buoyancy of all the detaching fluid must be positive, implying that all the potential energy released by negatively buoyant fluid is used for the entrainment of denser fluid. Both $V_{min}(R)$ and $V_{max}(R)$ increase monotonically with R, and so do the experimental values.

An example is given in Fig.3, which shows that the measured velocities lie comfortably within the predicted limits; in general the data lie much closer to V_{min} than V_{max}. Note also that as R→1 the fall velocity V of the interface is very much larger than the settling velocity of the smallest particles. Thus the rate of descent of the top of the sedimenting region can be greatly enhanced by the overlying convection, because of the wide spread of settling velocities of the sedimenting particles. The volume fraction of the particles carried upwards can also be calculated for the two limiting cases, as well as the unimpeded settling on the bottom. The theoretical predictions for the latter are seen to be in good agreement with the corresponding experimental values shown in Fig.2.

Extension to a Stratified Environment

Kerr (1991) has shown how these ideas can be applied to the case where there is a stable density gradient, instead of a constant ρ_U, in the region above an initially homogeneous sedimenting layer. In these circumstances the density gradient is eroded in a number of cycles of decreasing intensity and increasing duration. During each cycle some of the sediment settles to the base of the lower stagnant region in which there is unimpeded sedimentation, while the rest is carried up into a convecting region which encroaches on the overlying gradient to produce a well-mixed layer of increasing depth. When the top of the sharp sediment layer reaches the base of the tank, the source of buoyancy driving the convection is lost, and a more dilute, static layer of suspended particles remains. This state is similar to the initial condition, and so a further cycle of sedimentation in a layer with a sharp top, and convection above results. Eventually all the sediment settles, leaving a homogeneous

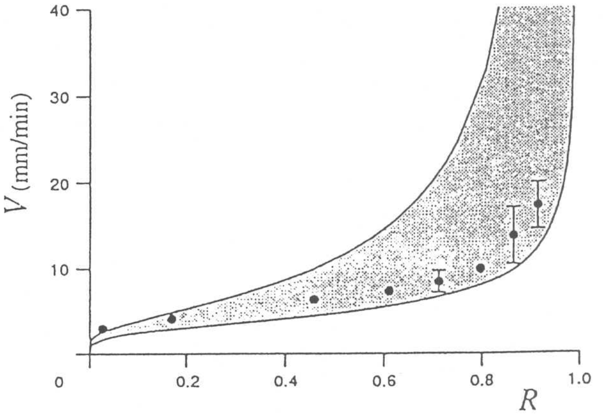

Fig. 3. The observed interfacial velocity as a function of the density ratio R, with ρ_B = 1.059, compared with the theoretical upper and lower bounds described in the text.

region of clear fluid with the remaining gradient above.

Kerr's (1991) theoretical explanation of these phenomena is based on the assumptions that the sediment interface velocity V is equal to V_{min} and that there is no density step between the convecting layer and the gradient above. He takes into account the time-dependent density ratio R(t), rather than assuming this is constant as it is with an upper layer of large volume. The agreement with the measured interface velocity at early times is good, though later $V > V_{min}$ as the convecting layer becomes deeper and the convection more vigorous. These results are relevant to the oceanic situation where a turbidity current formed (for example by a storm) in shallow regions carries lighter surface water down into the deep ocean. When the current stops, the light interstitial water can convect upwards to produce a thick layer of homogeneous deep water underlying a gradient. Such a structure has recently been observed in the Sulu Sea, as reported by Quadfasel et al. (1990).

MECHANICALLY MIXED SUSPENSION

During the flow of a turbidity current down a slope, the turbulence generated by the bottom stress is the dominant process keeping the sediment in suspension. Huppert et al. (1992) have studied a one-dimensional analogue of this system, using an oscillating grid to produce turbulence near the bottom of a tank. The results are contrasted with the corresponding case of a dense turbulent fluid layer entraining less dense fluid across a sharp interface.

Density-Stratified Fluid Layers

There have been many laboratory studies of turbulent mixing in stratified fluids without a mean flow (for recent reviews see Turner (1986) and Fernando (1991)). In most of these the turbulent motions have been generated by the vertical oscillation of a horizontal grid located in the interior of the stirred layer. With the application to turbidity currents in mind we have carried out experiments using a standard mixing box and grid (Turner 1986), but with the stirring grid located as close as possible to the bottom, at a mean height of z_0=1.9 cm above the base of the tank. We describe first the "control" experiments using density differences produced by dissolving sugar in the lower layer.

Experiments were carried out starting with a 6.0 cm layer of sugar solution and a range of initial density differences $\Delta\rho_0$, and various stirring frequencies ω. The depth h of the lower layer was recorded as a function of time t as it deepened due to the entrainment of the fresh water above. The observations, expressed in terms of the distance $z=h-z_0$ of the grid from the interface, are well fitted by a relationship of the form

$$z = Bt^b \,, \qquad (2)$$

where B is a function of ω and $\Delta\rho_0$ and b=0.151 ± 0.008. Previous investigations have expressed the rate of entrainment across the stable interface, v_E, in terms of the turbulent velocity scale at the interface

$$v_E = dz/dt = v_I f(Ri) \,, \qquad (3)$$

where f is a function of the Richardson number $Ri = g'z/v_I^2$ and $g' = g\Delta\rho/\rho_0$ is the reduced gravity.

Both the decay of turbulent velocity with distance from the grid and the effect of the resulting v_I on the entrainment across the interface need to be taken into account. Dimensional arguments suggest that

$$v_I = C_1 \omega d^{\alpha+1} z^{-\alpha} \text{ and } f(Ri) = C_2 Ri^{-\beta} \qquad (4a,b)$$

for some constants C_1, C_2, α and β, where d is a representative length specified by the geometry of the grid and its stroke. Substituting (4) into (3) and integrating, and comparing with the experimental relation (2), allows us to deduce the values of α and β, namely α=1.28±0.11 and β=1.696±0.072. These differ significantly from those previously obtained with the stirring grid located in the interior of the fluid, well away from the bottom boundary.

Experiments With a Sediment Layer

Next we present the results of experiments in which the tank and grid geometry were the same as the above, but the density increase in the lower layer was produced by adding silicon carbide grid, of density $\rho_p = 3.217 \text{gcm}^{-3}$. Four different batches of particles were used, with effective median diameters of 5.5, 5.8, 10.8 and 17.6μm as determined from the corresponding Stokes fall velocity v_s. A measured mass M_0 of particles suspended in water was added rapidly under a layer of fresh water, the grid was oscillated with frequency ω, and the system allowed to come to equilibrium. The frequency was then reduced in steps, and the new final states measured each time.

The time-history of the layer depth and the particle concentration are distinctly different from that of the sugar experiments, since the particles can migrate through and fall out of the stirred layer as it changes thickness, rather than simply contributing to the density difference. In particular, instead of deepening continuously according to (2), the layer behaviour changes greatly as ω and the intensity of turbulence is increased. This is illustrated in Fig.4 using data for three values of ω obtained in preliminary experiments using a smaller box and a different (non-standard) grid geometry. At the lowest stirring rate the particles eventually all precipitate while the layer depth first decreases, and then increases without limit. For an intermediate level of turbulence (ω=1.67) some particles remain in suspension after a long time, and the layer depth goes through a minimum before reaching an asymptotic steady state. With still higher stirring frequencies, all the particles are held in suspension in a layer which is slightly shallower and more concentrated than the initial distribution.

We concentrate here on the measurements of the final layer properties in the experiments where a steady state was attained (which often took several hours). A striking result is that the final thickness z_∞ of the suspension layer above the grid is very nearly proportional to ω. The multiplying constant is a function of both M_∞ and the particle size. As a guide to the most appropriate method of scaling the data, we now consider several alternative theoretical ideas. First, can the relations (4) with values of α and β deduced for the sugar

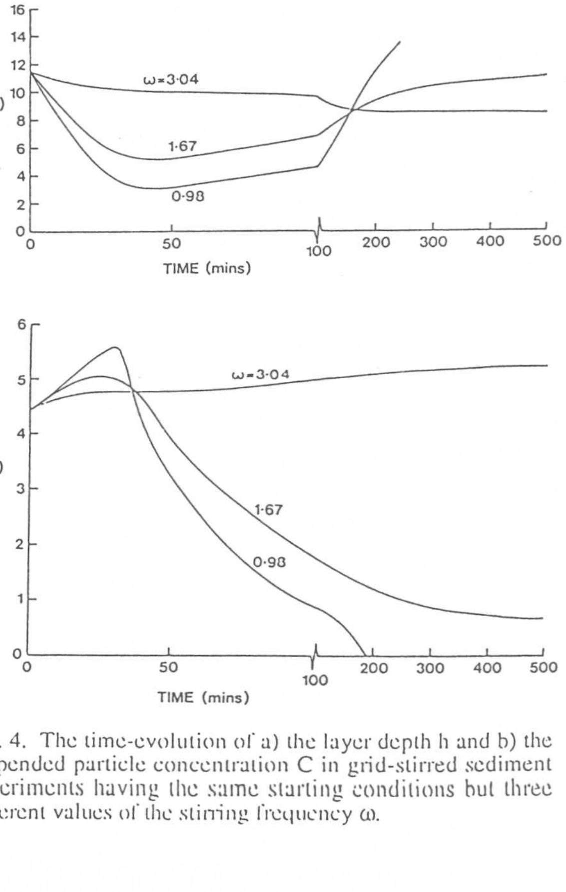

Fig. 4. The time-evolution of a) the layer depth h and b) the suspended particle concentration C in grid-stirred sediment experiments having the same starting conditions but three different values of the stirring frequency ω.

layer experiments in exactly the same geometry be carried across to suspensions? The extra assumption required is that the sediment layer stops growing when the median particle fall velocity v_s just equals the entrainment velocity v_E. This procedure does not lead to a satisfactory explanation of the data. It predicts that $z_\infty \propto \omega^{0.78}$, and if v_E is calculated for all the final layer depths for a given particle size it varies over a wide range, so it cannot be balanced by a fixed settling velocity v_s.

The following dimensional argument is equivalent physically to the more mechanistic theories which are based on the conservation of energy and include the effect of suspended particles on the whole of a sediment layer. The fluid motions near the oscillating grid are described by the parameter ωd^2, the grid action, where d is the lengthscale introduced in (4a) and ω is the frequency. There is a buoyancy flux associated with particles falling out equal to $g^* v_s C_\infty$, where C_∞ is the dimensionless volume concentration of particles, v_s is the median Stokes settling velocity and $g^*=g(\rho_p-\rho_o)/\rho_o$ incorporates the excess density of the particles relative to that of the fluid, ρ_o. When the mass of particles in the layer is steady, the upward particle (and buoyancy) flux driven by the turbulence must be equal to the downward flux due to settling. If ωd^2 and $g^* v_s C_\infty$ are the only two physically relevant parameters on which the final height z_∞ of the interface above the grid depends, it follows on dimensional grounds that

$$z_\infty = (\omega d^2)^{3/4}(g^* v_s C_\infty)^{-1/4}. \qquad (5)$$

Using the relation $M_\infty = A\rho_p C_\infty h_\infty$ between the "kinematic" load M_∞ of particles in suspension and the volume concentration C_∞, where A is the cross-sectional area of the tank, and taking due account of the difference between z_∞ and $h_\infty(=z_0+z_\infty)$, (5) can be rewritten as

$$z_\infty = (\omega d^2)(g^* v_s M_\infty / A\rho_p)^{-1/3}(1+z_0/z_\infty)^{1/3}. \qquad (6)$$

A plot of all the data, scaled according to (6), is shown in Fig.5. There is a very satisfactory collapse of the data onto a straight line, giving good support to the above theory once M_∞ is known. We still need to consider how M_∞ is related to the initial mass M_0 when only a fraction of the particles is held in suspension. A given mechanical energy input can support only a certain maximum particle concentration C_∞ and hence maximum buoyancy flux. If ω and hence the energy input are increased, we see that the general relation (6) is satisfied if there is an associated increase of M_∞, with z_∞ changing little. In fact Fig.5 contains data for a series of runs for which z_∞ remains approximately constant and M_∞ is proportional to ω^3, consistent with a balance between the kinetic energy input and the rate of increase of potential energy associated with lifting the particles.

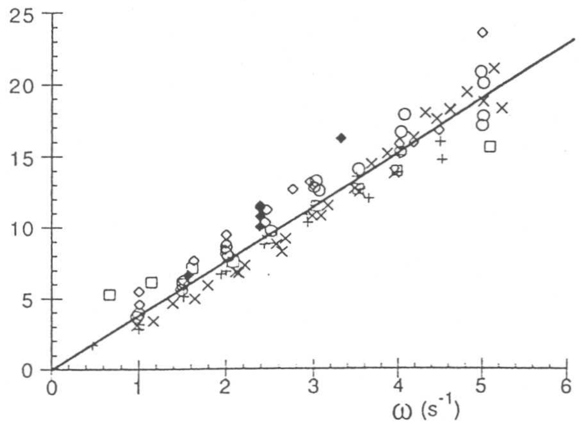

Fig. 5. All the measurements of layer depth for the grid-stirred experiments, with four different particle sizes and a wide range of M_∞, plotted according to (6), i.e. $E(M_\infty)=z_\infty(1+z_0/z_\infty)^{-1/3}(g^* v_s M_\infty/A\rho_p)^{1/3}$ is plotted as a function of ω. The collapse of the data onto the line gives good support to the theory.

SUMMARY AND DISCUSSION

We have presented laboratory observations and associated theoretical ideas which address two aspects of sediment-laden flows in nature. First we have considered sediment layers with light interstitial fluid, ponded under overlying fluid of density intermediate between the density of the bulk suspension and that of the interstitial fluid. Settling of the dense particles leads to the release of light fluid and vigorous convection in the upper layer, with unimpeded settling in the lower layer, below a sharp interface which descends at a constant velocity. This is applicable to brackish underflows in river deltas, turbidity currents and pyroclastic flows, once they have come to rest in a low lying region, but it does not consider the flowing gravity currents which set up this state.

The second study investigates the processes which maintain sediment in suspension in a turbulent gravity current. The turbulence generated by the bottom stress has been simulated in a tank with no mean flow, using an oscillating grid close to the bottom of a tank containing fluid of constant density plus suspended particles. We have determined the conditions for the maintenance of a steady sediment layer of fixed depth and concentration, and have shown that these are consistent with an energy argument relating the kinetic energy input and the buoyancy flux associated with the settling and resuspended particles.

We have carried out some preliminary experiments which combine the two processes described above, by setting up a stirred lower layer with light interstitial fluid. At one extreme there is convection in the upper layer and a static sedimenting layer below a descending interface; at the other we have a turbulent lower layer bounded by a sharp interface with no upward transport or turbulence above it. It will be of great interest to see if there is a steady or quasi-steady state where the two competing processes are producing turbulence on both sides of the interface.

Sparks et al. (1993) have carried out laboratory experiments on sediment-laden gravity currents, with lighter fluid holding the particles in suspension, and have examined the processes of sedimentation leading to eventual lift-off in that geometry. An associated theory has predicted reasonably well the observed deceleration of the currents and the lift-off distance. Much remains to be done, however, before the detailed mechanisms are fully understood, and there will be great experimental advantages in using grid stirring, rather than flow in a long channel, to produce the turbulence in the lower layer.

REFERENCES

FERNANDO, H J S (1991) Turbulent mixing in stratified fluids. Ann. Rev. Fluid Mech., 23, 455-493.

HUPPERT, H E, KERR, R C, LISTER, J R and TURNER, J S (1991) Convection and particle entrainment driven by differential sedimentation. J. Fluid Mech., 226, 349-369.

HUPPERT, H E, TURNER, J S and HALLWORTH, M A (1992) A laboratory study of mixing and sedimentation of a turbulent fluid suspension. Earth Planet. Sci. Lett. (submitted).

JONES, K P N, McCAVE, I N and PATEL, P D (1988) A computer-interfaced sedigraph for modal size analysis of fine grained sediment. Sedimentology, 35, 163-172.

KERR, R C (1991) Erosion of a stable density gradient by sedimentation-driven convection. Nature, 353, 423-425.

QUADFASEL, D, KUDRASS, H and FRISCHE, A (1990) Deep-water renewal by turbidity currents in the Sulu Sea, Nature, 348, 320-322.

SPARKS, R S J, BONNECAZE, R T, HUPPERT, H E, LISTER, J R, MADER, H and PHILLIPS, J (1993) Sediment-laden gravity currents with reversing buoyancy. Earth Planet. Sci. Lett. (submitted).

TURNER, J S (1986) Turbulent entrainment: the development of the entrainment assumption, and its application to geophysical flows. J. Fluid Mech., 173, 431-471.