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The Crystallization of Lava Lakes

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Thermal convection driven by the kinetic undercooling of solidification can produce compositional and textural variations in an initially uniform lava flow that is cooled predominantly from above. We show that such kinetic undercooling is sufficient to drive convection at high Rayleigh numbers even in shallow lava lakes. We then employ a theoretical model that includes the effects of convection at high Rayleigh number to make predictions of the complete evolution of the lava. We show that predictions of the rate of growth of solid crust at the surface of the lake do not differ much from predictions made by ignoring effects due to convection. However, convective motions do shorten the time for complete solidification because, when they are driven by kinetic undercooling, there is internal crystallization in addition to that occurring near the cooled upper boundary of the lava. This important effect, which is absent from purely conductive models, predicts stratification and zonation of the lava, in agreement with field observations. We also examine the effect of taking into account the heat transfer to the country rock below the lava lake, and determine how the evolution of the lava lake varies with the viscosity of the lava.

INTRODUCTION

A recent study of the crystallization of magma chambers cooled from above [*Worster et al.*, 1990] has shown how thermal convection, coupled with the non-equilibrium kinetics of crystallization, can cause zonation of the mineral constituents and preferential crystal accumulation at the chamber floor. In this paper we show that the kinetic undercooling adjacent to the solidification front is sufficient to drive convection with moderate to large values of the Rayleigh number in lava lakes. The driving physical process is the solidification of minerals at the top of the lava lake caused by cooling through its upper boundary with overlying air or seawater. We suppose that heat transfer to the air or water is sufficiently efficient to maintain the upper boundary at a fixed temperature. The growth of crystals in the upper margin is accounted for by a detailed model of partially solidified zones (mushy layers) that has been well tested against laboratory experiments.

Kinetic undercooling at the interface of the mushy region with the fluid lava below leads to thermal Rayleigh numbers that are sufficient in most cases to drive thermal convection throughout the lava. This causes the lava to become undercooled in its interior, which allows further crystallization to take place remote from the cooled upper boundary in the interior or at the floor of the lava lake. It is this internal crystallization and the accompanying sedimentation and/or compositional convection that can cause differentiation within the lava. A lava lake is cooled strongly at its upper surface, which is in contact with air or water, and less strongly at its base by conduction through the underlying country rock. We begin by ignoring any heat transfer through the base in order to focus attention on consequences that are solely in response to cooling at the roof. We then determine the enhancement of solidification at the bottom of the lake caused by heat transfer to the country rock.

In most of our calculations we use physical parameters that correspond to mixtures of diopside and anorthite, primarily because this is a simple system for which all the relevant parameters are known. However, this is a somewhat low-viscosity silicate system compared to common magma types, so we include in this study an investigation of the effects of increasing the viscosity of the melt. We compare the results with previous theoretical models of lava lakes and with observations of the cooling and differentiation of Hawaiian lava lakes.

A MODEL LAVA LAKE

We imagine that lava has rather quickly filled a depression on the surface of the earth to form a lake, which we suppose to have an initially uniform composition C_0 and temperature T_0. The lava begins to cool at its base by conduction into the country rock and at its roof by convective heat transfer to the air or water (if it is a submarine flow) above it. We shall assume that the boundary of the lava with the overlying air or water is maintained at a fixed temperature T_b and that the temperature of the country rock far beneath the flow is also T_b. Few lava flows are superheated when they are erupted, and *Turner et al.* [1986] have shown that, in any case, convection in a magma cooled from above quickly removes any initial superheat that might exist. Lava lakes on Hawaii contain some phenocrysts on formation [*Wright et al.*, 1976] and are probably undercooled as a result of degassing during their eruption. In addition, considerable convective foundering of cooled crust takes place before a stable solid crust has formed. For simplicity, however, we assume herein that the lava is initially at its liquidus temperature $T_0 = T_L(C_0)$.

We assume that the lateral dimensions of the lake are much greater than its depth and study the one-dimensional problem illustrated in Figure 1. As heat is extracted from the upper margin of a lava lake, a solid crust of composite solid is formed.

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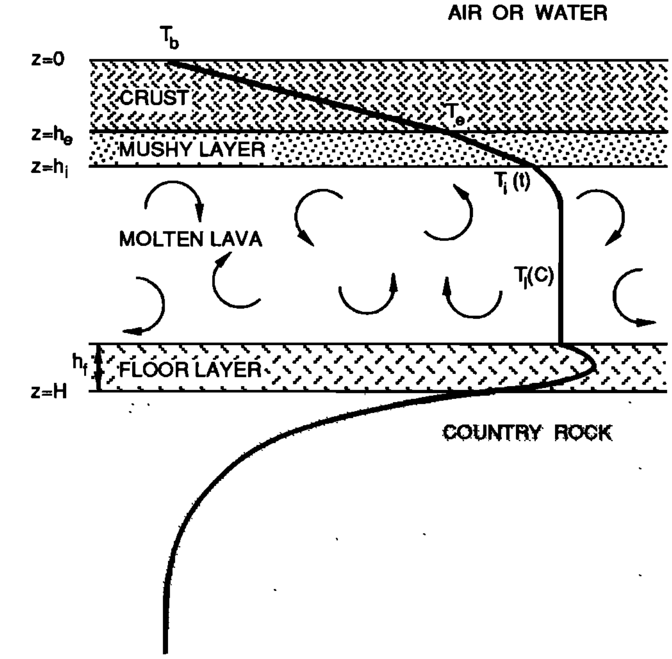

Fig. 1. Schematic diagram of a lava lake that is being cooled at its surface by contact with the cold air or water above it and also cooled by conductive heat transfer to the country rock beneath. A crust of composite solid grows downward from the roof where the temperature T_b is constant. A mushy layer separates the composite solid from the molten lava, which is convecting vigorously. Internal crystallization, which occurs as a result of the interaction of convection with the kinetic undercooling at the interface with the mushy layer, forms a solid layer near the floor of the lake. A sketch of the temperature field is indicated on the diagram.

Below this crust, where the temperature is greater than the solidus temperature, a mushy layer can form comprising a connected matrix of crystals and interstitial melt. The interface between the mushy layer and the molten lava beneath it has a temperature T_i that is approximately equal to but slightly below the equilibrium freezing temperature $T_L(C)$ of the molten lava. It is this temperature difference that drives convection in our model and causes the region of molten lava to become undercooled. We assume that any internal crystallization that occurs in response to this undercooling either takes place at the bottom of the lake or forms suspended crystals that eventually settle to the floor. Additional cooling and crystallization may take place at the floor in response to the conduction of heat to the country rock below.

Initially we shall ignore the heat transfer at the floor entirely in order to focus attention upon phenomena that occur solely in response to the cooling at the roof. We also ignore other effects, such as degassing, which may have significant influence on the behavior of real lava lakes [*Wright et al.*, 1976; *Wright and Okamura*, 1977; *Helz*, 1987]. We discuss later how these other effects might modify the behavior of lava lakes.

The dominant thermal balance governing the growth of the crust and mushy layer is between conduction to the upper boundary and the latent heat that must be removed in order to effect the change of phase from liquid to solid. A straightforward scaling analysis based on this balance, without consideration of convective effects, gives the depth of crust and mushy layer as

$$h \sim \left((T_L - T_b)C_p\kappa/L\right)^{1/2} t^{1/2}, \tag{1}$$

where L and C_p are the latent and specific heat, respectively, and κ is the thermal diffusivity. Detailed analyses of the growth of the crust have been made [*Carslaw and Jaeger*, 1959; *Peck et al.*, 1977; *Turcotte and Schubert*, 1982] on the assumption that conduction is the sole means of heat transfer in the system. *Provost and Bottinga* [1972] included some effects of convection by assuming a well-mixed interior of uniform temperature which decreased linearly with time as the crust grew. Their model, however, did not incorporate any coupling between convection and kinetic effects of crystallization. A certain (small) amount of kinetic undercooling always exists at an advancing solid–liquid interface and this can be sufficient to cause convection of the molten lava in the interior of the lake [*Brandeis and Jaupart*, 1986; *Kerr et al.*, 1989; *Worster et al.*, 1990]. The kinetic undercooling in solidifying, pure diopside has been measured by *Kirkpatrick et al.* [1976], who give an expression of the form

$$\frac{dh}{dt} = \mathcal{G}\mu^{-1}(T_L - T_i)^2 \tag{2}$$

relating the growth rate dh/dt of a crystal interface to the level of interfacial undercooling $T_L - T_i$, where μ is the dynamic viscosity of the melt and $\mathcal{G}$ is a constant. The kinetic undercooling measured by *Kirkpatrick et al.* [1976] was associated with single crystals of pure diopside. We use the same formula here for diopside crystals in a mushy layer growing into a melt of diopside and anorthite. The edge of the mushy layer is an envelope of the tips of crystals, and it is appropriate to use the kinetic law for a single crystal to determine the rate of advance of the envelope. However, in addition to kinetic undercooling, there is undercooling at the edge of a mushy layer associated with a compositional boundary layer there. In the present context such compositional undercooling is estimated to be much smaller than the kinetic undercooling, and we therefore neglect it. Comparing (2) with the scaling given by (1) shows that the interfacial undercooling has a typical magnitude of

$$T_L - T_i \sim \sqrt{\frac{(T_L - T_b)C_p}{L}\frac{\mu\kappa}{\mathcal{G}H}}, \tag{3}$$

where H is the total depth of the lake.

When a melt is cooled and crystallized, there is usually a temperature gradient set up in the melt ahead of the solidification front, as measured, for example, by *Huppert and Worster* [1985] and by *Tait and Jaupart* [1992]. When the cooling takes place at an upper boundary of the melt, this temperature gradient can cause convection in the melt [*Turner et al.*, 1986; *Kerr et al.*, 1990*a*]. In these former studies the temperature gradient was a consequence of the melt being superheated and the phase boundary being at the equilibrium freezing temperature. In the case being studied here, in contrast, the temperature gradient is a result of the difference between the equilbrium temperature of the melt and the undercooled temperature at the interface. The Rayleigh number

$$Ra = \frac{\alpha g(T_L - T_i)H^3}{\kappa\nu}$$

based upon this undercooling and the depth of the lava lake has magnitude

$$Ra \sim \frac{\alpha g}{\nu}\sqrt{\frac{\mu}{\kappa\mathcal{G}}\frac{(T_L - T_b)C_p}{L}}H^{5/2}, \tag{4}$$

where α is the coefficient of thermal expansion, ν is the kinematic viscosity and g is the acceleration due to gravity. For typical order-of-magnitude values of the physical parameters, as given in Table 1, (4) shows that the Rayleigh number Ra is between 10^8 and 10^{12} when H is in the range 3–100 m. Thus, in the absence of other effects, the lava will convect vigorously even though the driving temperature difference $T_L - T_i$ is only between about 6°C and 1°C. This Rayleigh number is larger for deeper lakes, although the driving temperature difference is smaller. Note that, even though the viscosity of magmas is highly temperature-dependent, it is appropriate to use a Rayleigh number based on constant viscosity in this case since the temperature difference across the boundary layer driving convection is very small.

Expression (4) is used to determine the depth of lava lake required to produce a Rayleigh number of 10^9 in terms of the viscosity of the lava, for various values of the kinetic coefficient $\mathcal{G}$. This relationship is illustrated in Figure 2. We see that for the Di–An system the typical Rayleigh number exceeds 10^9, provided that the lava lake is deeper than about 6 m. On the assumption that pyroxenes and plagioclase growing in a basaltic melt have similar values of $\mathcal{G}$, but that the viscosity is 30 times larger, the corresponding value for a basaltic lava lake is about 12 m as shown in Figure 2.

With these parameter values the Reynolds number of the convective flow is between about 0.5 and 10, so that viscous stresses are significant. However, the Rayleigh number is well above the critical value (of approximately 10^3) required for the thermal boundary layer near the cooler upper boundary of the region of melt to be convectively unstable, and the temperature field will be highly disordered and well mixed over time scales comparable to the total solidification time. The Peclet number is between approximately 600 and 10^4 so that convective heat transfer dominates conductive heat transfer in the interior of the melt. An alternative argument that leads to the same conclusion was presented by *Kerr et al.* [1989] who showed that the time taken for breakdown of the thermal boundary layer at the mush–liquid interface is typically much shorter than the time for complete solidification (as discussed, for example, by *Brandeis and Jaupart* [1986]. For example, in our model the time scale of breakdown of the thermal boundary layer is a few hours for a 100 m layer, which takes about 35 years to solidify completely.

A mathematical model incorporating the effects of convection driven by kinetic undercooling was developed by *Kerr et al.* [1989, 1990a, b, c], and used by *Worster et al.* [1990] to analyze the cooling and solidification of magma chambers. We refer the reader to these publications for details of the model and concentrate here on the results of computations made using the same governing equations, although with different boundary conditions. The principal difference between the calculations presented here and those of *Worster et al.* [1990] are that here we hold the temperature at the top of the lava lake fixed, rather than having a decaying temperature profile in the country rock above a magma chamber. We again restrict our attention to lavas having compositions in the Di–An system, since it is for such lavas that we have the most comprehensive data on physical properties, including the kinetic coefficient $\mathcal{G}$.

TABLE 1. The Order-of-Magnitude Parameter Values Used to Estimate the Depths of Lava Lakes of Various Viscosities Required to Give a Rayleigh Number of 10^9 in Figure 2

Quantity	Value	Units
α	10^{-4}	$°C^{-1}$
g	10^3	$cm\ s^{-2}$
κ	8×10^{-3}	$cm^2\ s^{-1}$
$T_L - T_b$	10^3	°C
C_p	0.4	$cal\ g^{-1}\ °C^{-1}$
L	10^2	$cal\ g^{-1}$
ρ	2.6	$g\ cm^{-3}$

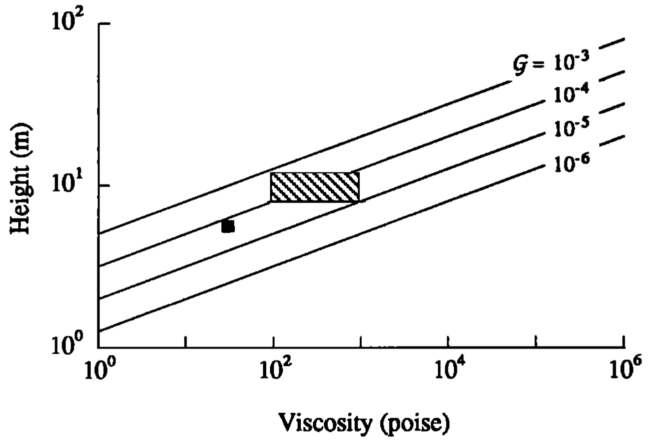

Fig. 2. The depth of lake required in order that the typical Rayleigh number is 10^9 for a lava of given viscosity. The different lines indicate how this critical depth varies with the kinetic coefficient $\mathcal{G}$. The filled square corresponds to the viscosity and kinetic coefficient of the Di–An system, and shows that the Rayleigh number exceeds 10^9 if the lava lake is deeper than about 6 m for this system. The hatched rectangle spans the typical viscosities of basalt.

The model can be described briefly as follows. The thermal diffusion equation is solved in the composite solid region with boundary conditions $T = T_b$ at the top surface of the lake and $T = T_e$ at the eutectic front $z = h_e$. The rate of advance of the eutectic front is determined by a Stefan condition, which balances the release of latent heat at the front with the difference in the conductive heat fluxes on either side of the front. The equations describing the evolution of the mushy region are essentially diffusion equations modified by the internal release of latent heat by the growing crystals. The rate of advance of the mush–liquid interface is determined by the kinetic condition (2), while the Stefan condition is used to determine the interfacial temperature T_i. This condition can be expressed as

$$k_m \frac{\partial T}{\partial z}\bigg|_{h_i-} = F_T + (T_l - T_i)\frac{dh_i}{dt}, \qquad (5)$$

where k_m is the conductivity of the mushy layer, T_l is the temperature of the melt, and F_T is the convective heat flux from the melt to the mushy layer. The heat flux

$$F_T = \gamma k_l \left(\frac{\alpha g}{\kappa_l \nu}\right)^{\frac{1}{3}} (T_l - T_i)^{\frac{4}{3}} \qquad (6)$$

is caused solely by the kinetic undercooling $T_l - T_i$. In addition to retarding slightly the growth of the mushy layer, this heat flux serves to cool the interior of the lava lake. In the detailed calculations to be described, all the physical parameters are taken as constants with the exception of the viscosity $\nu(T)$, which is a function of the temperature of the melt.

Internal Crystallization

As the melt cools, further crystallization can take place in the interior of the lava lake. A good estimate of the volume of crystals formed in this way can be made using the following conservation equations for heat and solute.

To begin with, we arbitrarily assume that there is no heat loss from the lava lake to the country rock below it. This allows us to determine what features of the results are determined solely by the cooling at the roof. In the case of negligible heat loss through the boundaries of the lava lake other than its upper surface, conservation of heat requires that

$$F_T = \rho_s L_s \frac{dh_f}{dt} - \rho_l C_{pl}(H - h_i - h_f)\frac{dT_l}{dt} - \rho_s C_{ps} h_f \frac{dT_l}{dt}. \quad (7)$$

In this study, h_f is treated as the depth of a layer of solid of zero porosity forming at the bottom of the lake due to internal crystallization. However, the same equation applies regardless of where the internal crystallization takes place as long as h_f is interpreted as V/A, where V is the volume of crystals grown internally and A is the horizontal area of the lake. An assumption made in obtaining (7) is that the temperature of the crystals is spatially uniform and equal to the temperature T_l of the melt. The heat flux F_T transferred to the upper crust of the lake balances the three terms on the right-hand side of (7), which represent, respectively, the latent heat of formation of the internal crystals, the loss of sensible heat from the melt, and the loss of sensible heat from the crystals.

Conservation of solute in the region below the crust requires that

$$(C_s - C_l)\frac{dh_f}{dt} = -(H - h_i - h_f)\frac{dC_l}{dt}, \quad (8)$$

where C_s is the composition of the internal crystals and C_l is the composition of the melt.

We estimate the rate of internal crystallization by assuming that it is sufficient to keep the interior in thermodynamic equilibrium, so that

$$T_l = T_L(C_l) \quad (9)$$

where $T_L(C)$ is the liquidus relationship.

Equations (7), (8), and (9) can be combined to give

$$\left[\rho_s L_s + \frac{C_s - C_l}{C_l'(T_l)}\left(\rho_l C_{pl} + \rho_s C_{ps}\frac{h_f}{H - h_i - h_f}\right)\right]\frac{dh_f}{dt} = F_T \quad (10)$$

from which the rate of internal crystallization dh_f/dt is determined.

The model presented so far is identical to that of *Worster et al.* [1990] except that here the top of the lava lake is held at a fixed temperature, whereas in the earlier model, relevant to the solidification of a magma chamber, heat was conducted through the country rock above the chamber.

RESULTS

We begin by showing the results of calculations for a lava lake 30 m deep consisting of pure diopside. It is assumed to have been formed at its liquidus temperature of 1392°C, and the temperature of its upper surface is held constant at 0°C. Figure 3 shows the history of solidification according to two different models. The first prediction (dashed curve) is made by assuming that conduction is the sole means of heat transfer in the system. The figure shows that solidification occurs only in a solid crust that grows downward from the cooled upper surface of the lake and that the lake is completely solidified after approximately 9 years. The solid curves show the results of a model in which the effects of vigorous convection in the melt are taken into account. Notice that the predictions for the rate of growth of the crust are similar in both calculations, disagreement between the two being rather less than the typical uncertainties in current geological measurements. Therefore field observations of the growth of crust at the upper surface of a lava lake provide a poor diagnostic for determining whether or not the underlying molten lava is convecting. For example, *Peck et al.* [1977] obtained a good fit to data on temperature as a function of depth for the 13.7-m-thick Alae lava lake using a totally conductive cooling model and adjusting the thermal properties to give the best fit to the data. By considering slightly different choices of the thermal conductivities of the crust and country rock than we have taken here, we could produce growth curves for the roof sequence that are barely distinguishable from the results obtained assuming convective cooling.

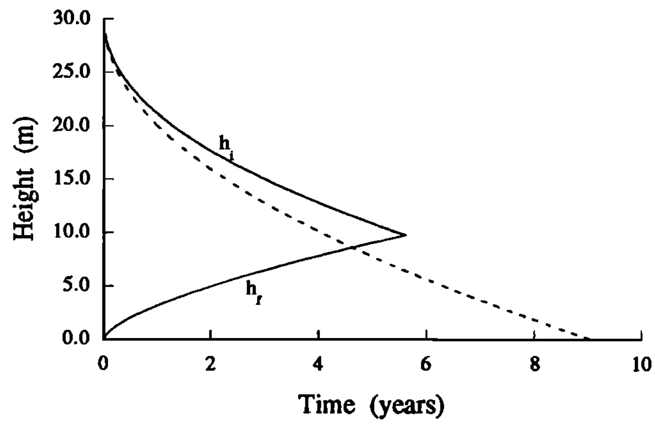

Fig. 3. The growth of solid as a function of time for a lava lake of pure diopside of total depth 30 m. The lava has an initial, uniform temperature of 1392°C, equal to its melting temperature, and the surface temperature of the lake is held constant at 0°C. The dashed curve shows the predictions of a calculation in which convection of the lava is ignored. The solid curves show the predictions of a more realistic calculation in which the heat transfer due to vigorous convection is included. Solid grows in the interior of the lake to form a layer of depth h_f at the base of the flow in addition to a layer of thickness h_i attached to the roof.

On the other hand, the second model predicts that some 30% of the lava crystallizes in the interior or at the floor of the lake even though, by assumption, the lava is being cooled only at its upper surface. This prediction is a consequence of the coupling of the kinetic undercooling associated with crystallization and the fluid motions caused by thermal buoyancy.

When the lava consists of more than one mineral component, compositional variations are developed in the flow. Some typical results are illustrated in Figure 4, which shows the time history of solidification and the final compositional profile in lava lakes of three different initial depths. In these and all the results henceforward, the data given in Table 2 were used. The melt layer initially had uniform composition $Di_{80}An_{20}$ and temperature equal to its liquidus temperature 1352°C, in each case. In a binary system such as the one studied here, there are two solidifying regions near the cooled boundary. Closest to the boundary there is a completely solid region comprising a composite of crystals of both end members of the binary system. In this region the temperature is below the eutectic temperature of the system. Below the solid region is a mushy layer in which crystals of one component (here diopside) are bathed in interstitial melt.

We assume that the crystals are firmly attached to the crustal roof of the lava lake. In previous studies of melting in magma chambers [*Huppert and Sparks*, 1988] the crystals were implicitly assumed to be freely suspended and could therefore

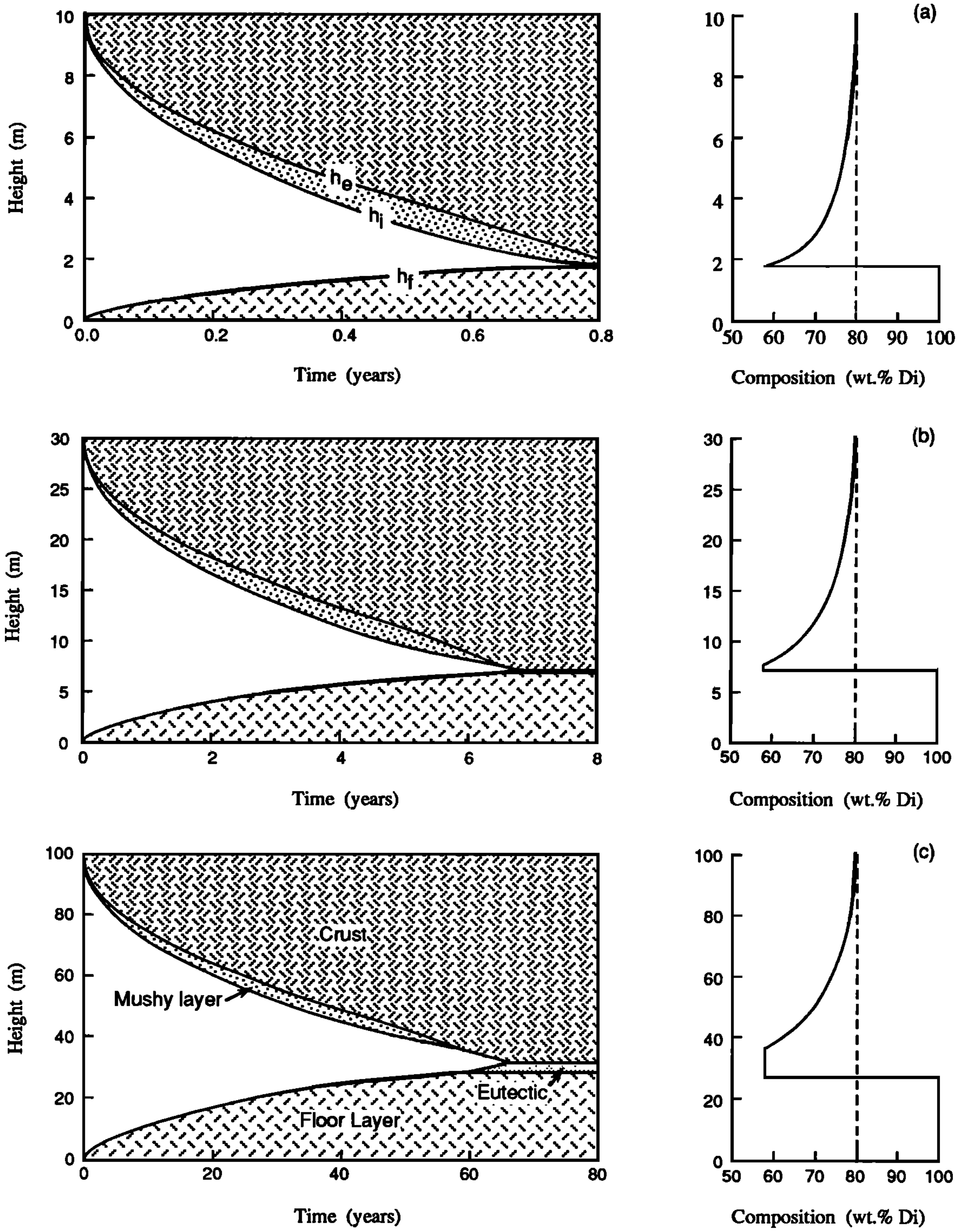

Fig. 4. History of solidification and final compositional profile in lava lakes of initial composition $Di_{80}An_{20}$ and initial temperature equal to the liquidus temperature of 1352°C. The curves labelled h_e, h_i, and h_f show the interfaces between the crust and the mushy layer, the mushy layer and the melt, and the melt and the floor layer, respectively, as sketched in Figure 1. The three diagrams indicate the different evolutions experienced by intrusions of different initial depths H (*a*) $H = 10$ m. (*b*) $H = 30$ m. (*c*) $H = 100$ m.

take part in convection. We note here that in such a case the dependence of the viscosity of the lava on both the temperature and the local crystal fraction should be taken into account, as described, for example, by *Davaille* [1991]. It can then be argued [*Jaupart and Parsons*, 1985] that only the lower edge of this mushy region forms a mobile slurry which convects along with the underlying melt. In the present study, we consider a case in which none of the mushy region is mobile. The temperature difference across the thermal boundary layer below the mushy layer is then quite small so that variations in viscosity need not be considered.

Figure 4 shows that the mushy region is relatively thin compared with the composite solid region, which is due to the fact that the difference between the liquidus temperature of the melt and the eutectic temperature is small relative to the difference between the eutectic temperature and the tempera-

TABLE 2. Explicit Parameter Values Used in Calculations of the Solidification of Diopside-Anorthite Lava Lakes

Quantity	Value	Units
ρ_α	2.65	g cm^{-3}
ρ_β	2.69	g cm^{-3}
ρ_l	2.60	g cm^{-3}
ρ_r	2.60	g cm^{-3}
Cp_α	0.274	cal g^{-1} °C^{-1}
Cp_β	0.273	cal g^{-1} °C^{-1}
Cp_l	0.380	cal g^{-1} °C^{-1}
Cp_r	0.380	cal g^{-1} °C^{-1}
k_α	0.008	cal cm^{-1} s^{-1} °C^{-1}
k_β	0.008	cal cm^{-1} s^{-1} °C^{-1}
k_l	0.008	cal cm^{-1} s^{-1} °C^{-1}
k_r	0.008	cal cm^{-1} s^{-1} °C^{-1}
L_α	94	cal g^{-1}
L_β	159	cal g^{-1}
α	1.1×10^4	°C^{-1}
g	980	cm s^{-2}
γ	0.14	
$\mathcal{G}$	5.6×10^{-5}	cm s^{-1} °C^{-2} poise
μ	$\exp[(12 - 52.5x + 62.5x^2)\ln 10]$; $x = 1 - 1000/(T + 273)$	poise
ν	$1000\exp[(1.5 + (1345 - T)/122)\ln 10]/\rho_l$	cm^2 s^{-1}

The physical parameters are the density ρ, the specific heat Cp, the conductivity k, the thermal expansion coefficient α, the acceleration due to gravity g, the coefficient γ in the relationship for the convective heat flux, and the kinetic coefficient $\mathcal{G}$. The dynamic viscosity μ, used in the determination of the kinetic growth law, and the kinematic viscosity of the melt ν are given as functions of the temperature of the melt T, expressed in degrees Celcius. The subscripts α refer to properties of anorthite, β to the properties of diopside, l to those of the liquid melt, and r to those of the country rock.

ture of the cooled upper boundary. The compositional profiles in Figure 4 show that the roof sequence has decreasing concentration of the primary solidifying phase (diopside) with depth. In deeper lava lakes (Figures 4*b* and 4*c*) the roof sequence can vary all the way to the eutectic concentration, in which case the central portion of the solidified lake has uniform composition equal to the eutectic.

The two principal effects of convection in the theoretical model are illustrated further in Figure 5. Figure 5*a* shows the dimensionless time $\kappa t_*/H^2$ for complete solidification as a function of the initial depth of the lava lake H. If conduction were the sole means of heat transfer then this dimensionless solidification time would be constant (equal to about 0.203). That it decreases as H increases demonstrates the increasing importance of convection as the Rayleigh number, which is proportional to $H^{5/2}$, increases. The stronger convective heat transfer from the interior of the lava lake to the roof enhances the internal growth of crystals relative to the growth of the crust at the roof. This is demonstrated in Figure 5*b*, which shows that the proportion of solid formed in the interior or at the floor of the lake increases as the overall depth of the lake increases. In these figures we also show the values determined by *Worster et al.* [1990] for magma chambers. The only difference between the calculations is in the boundary conditions applied to the top of the system, as discussed previously. Keeping the top of the lava lake at a fixed temperature decreases the cooling time by a factor of about two and decreases slightly the proportion of crystals grown internally.

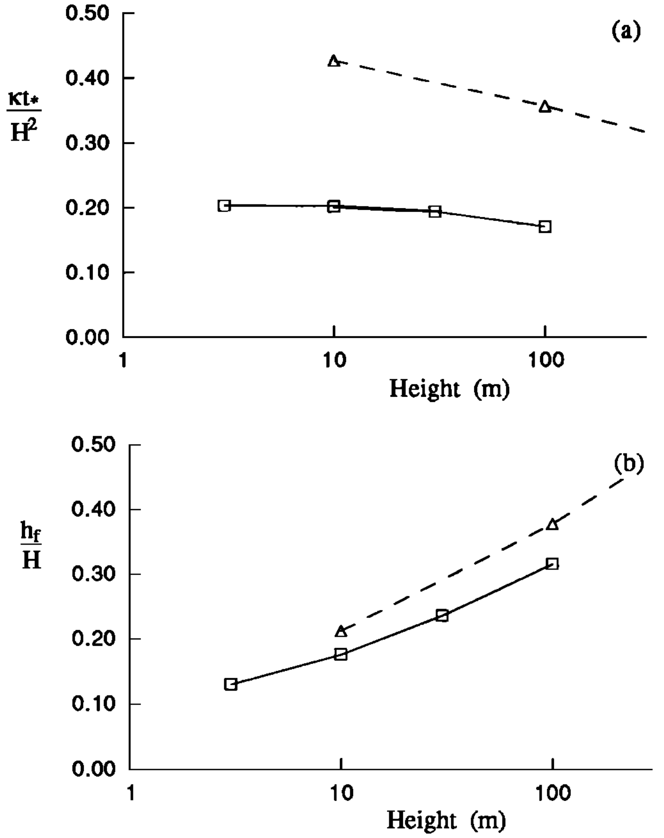

Fig. 5. (*a*) The ratio $\kappa t_*/H^2$ as a function of the initial depth of the lava lake H, where t_* is the time taken for complete solidification. This ratio would be constant if the heat transfer were purely conductive. The ratio decreases as H increases and convective heat transfer becomes more important. (*b*) The ratio of the final depth of the layer formed by internal crystallization h_f to the total depth H of the lava lake as a function of H. The solid lines show the results for lava lakes in which the top of the lake has a fixed temperature. The dashed lines show the results for magma chambers, which lose heat by conduction into the overlying country rock [*Worster et al.* 1990, corrected by *Huppert and Worster*, 1992].

Figure 6*a* illustrates the typical evolution of the interior temperature of the lava T_L and the corresponding temperature T_i at the interface between the molten lava and the mushy layer. Despite the fact that the temperature difference $T_L - T_i$, which drives the convection in the lava, is only a few degrees centigrade, the associated Rayleigh number remains large throughout most of the solidification history, as shown in Figure 6*b*. The rapid decrease in the Rayleigh number near the end of its evolution is due primarily to the shrinking depth of the molten portion of the lava.

HEAT LOSS THROUGH THE FLOOR

We see from (10) that in the model analyzed so far, crystals grow internally solely in response to the convective heat transfer F_T to the upper surface of the lava lake. All the latent heat must ultimately be transferred by conduction through the upper crust of the lake to the air or water above the lake. It is clear that the rate of internal crystallization will be enhanced if some of this latent heat is transferred to the country rock beneath the lake. We can take this into account by solving the thermal diffusion equation in the country rock and floor layer. In this case it is necessary to assume that any crystals that form in suspension settle to the floor of the lake rapidly

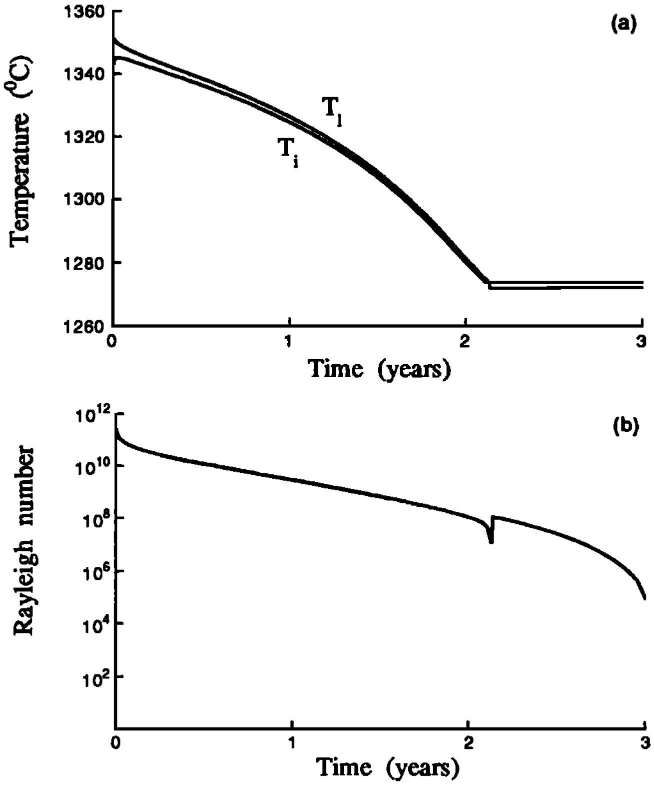

Fig. 6. (a) The temperature of the molten lava T_l and of the interface between the mushy layer and the molten lava T_i as functions of time for the conditions of Figure 4b. (b) Even though T_i is only slightly less than T_l, the difference is sufficient to drive vigorous convective motions within the melt, as indicated by the very large values of the Rayleigh number, shown here as a function of time. The Rayleigh number remains large except right near the end of the evolution when the depth of remaining molten lava becomes small. The small blip at about 2 years is a consequence of the different growth rates that ensue once the eutectic front has caught up with the edge of the mushy layer.

compared with the overall rate of solidification. Equation (7) is replaced by

$$F_T = \rho_s L_s \frac{dh_f}{dt} - \rho_l C_{pl}(H - h_i - h_f)\frac{dT_l}{dt} - k_s \frac{\partial T}{\partial z}\bigg|_{z=h_f-} \qquad (11)$$

so that now F_T is balanced by the release of latent heat, the loss of sensible heat from the melt, and the heat transferred through the floor layer to the country rock. Combining (11) with (8) and (9) yields

$$\left[\rho_s L_s + \rho_l C_{pl} \frac{C_s - C_l}{C_l'(T_l)}\right] \frac{dh_f}{dt} = F_T + k_s \frac{\partial T}{\partial z}\bigg|_{z=h_f-}. \qquad (12)$$

If internal crystallization occurs principally due to cooling at the roof, then the conductive heat flux at the top of the floor layer $\mathcal{F} = k_s \frac{\partial T}{\partial z}\big|_{z=h_f-}$ is negative and this term in (12) serves to retard the growth of the floor layer. It plays the same physical role as the term expressing the sensible heat of the basal layer in (10). On the other hand, if the cooling through the floor is strong enough, then $\mathcal{F}$ is positive and the secondary crystallization is driven by this cooling as well as by the cooling through the roof. Note that according to (10) $dh_f/dt = 0$ when $F_T = 0$, while according to (12), growth of crystals internally continues even when $F_T = 0$.

The growth of crystals internally is accompanied by compositional convection. The dynamics of this convection are not addressed in this paper. Rather the fluxes of heat and solute from the floor into the interior of the lake are inferred from heat and solute conservation coupled with the assumed constraint of thermodynamic equilibrium. This approach was first suggested by *Kerr* [1984] and used by *Woods and Huppert* [1989] to analyze an experiment in which an aqueous solution of sodium carbonate was solidified by cooling from below. A predictive model of the dynamics of convection driven by the release of solvent during solidification has yet to be devised, although Woods and Huppert made several suggestions for how the heat and solute fluxes might couple dynamically, and *Huppert* [1990] has derived a general form of the relationship between the heat and solute fluxes from dimensional analysis.

With this shortcoming in mind we can nonetheless gain some feeling for the significance of additional cooling to the country rock using the present model. The country rock is assumed to be initially at 0°C. Once the lava has formed a lake above the country rock, the latter begins to warm up as heat is transferred from the lava. The decaying temperature profile in the country rock provides a time-varying (decreasing) heat flux out of the lava lake into the country rock.

Some results are illustrated in Figure 7, which shows the growth of the various solidifying regions and the final compositional profile in a lava lake 100 m deep. This figure should be compared with Figure 4c. It is notable that the addition of cooling through the floor shortens the time for complete solidification by a factor of about two. This shows that the total heat transfer to the country rock is significant even though the heat flux is decaying with time. The depth of the floor layer is increased substantially relative to the thickness of the upper crust, and the associated increase in release of buoyant fluid at the floor causes the composition of the roof sequence to vary more rapidly with depth. Further comparisons between the predictions made with and without bottom cooling are shown in Figure 8, which shows the dimensionless time for complete solidification and the relative depth of the floor layer as functions of the initial depth of the lake in each case. Also included for comparison are results taken from *Worster et al.* [1990] for the cooling of a magma chamber that is insulated at its base.

THE EFFECT OF VISCOSITY

The diopside-anorthite system has a viscosity lower than that of common lava types. For example, basaltic lavas typically have viscosities 10 to 30 times larger. In order to gain some feeling for the importance of the viscosity of the melt in determining the evolution of the solidifying lake, we made some calculations using all the parameter values of the Di–An system but multiplying the kinematic viscosity ν of the melt by constant factors. The only element of the model affected by this change is the convective heat flux F_T, which is proportional to $\nu^{-1/3}$ and hence decreases as ν increases.

Figure 9 shows the depth of solid formed internally (at the base) for melts of different kinematic viscosity. We see that if there is negligible heat loss through the bottom of the lake, then the amount of internal crystallization is significantly reduced in lavas of high viscosity. The range of viscosities of basalt are indicated on the diagram. Once the viscosity is 10^6 times that of the Di–An system, the model evolves almost as if convection were ignored. It should be noted, nevertheless, that the Rayleigh number of the lake is still about 10^{10} in this

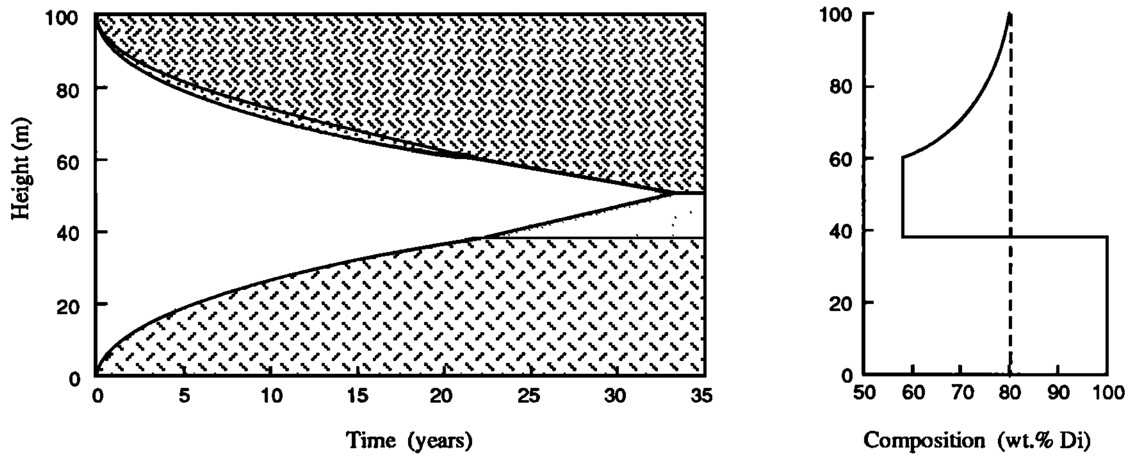

Fig. 7. The evolution of a 100-m-deep lava lake of composition $Di_{80}An_{20}$ when it is cooled by conduction to the country rock below, in addition to being cooled at its upper surface. This figure should be contrasted with Figure 4*c*, which shows the evolution of a similar lava lake when conduction to the country rock is neglected.

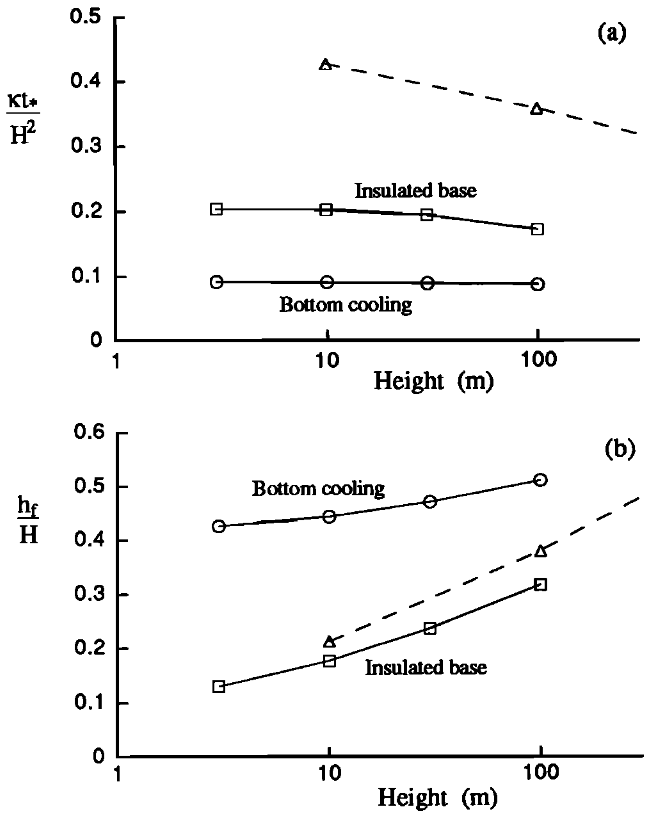

Fig. 8. Results obtained with and without taking into account the heat transferred to the country rock. (*a*) The ratio $\kappa t_*/H^2$ as a function of the initial depth of the lava lake H, where t_* is the time taken for complete solidification. This ratio would be constant if the heat transfer were purely conductive. The ratio decreases as H increases and convective heat transfer becomes more important. (*b*) Ratio of the final depth of the layer formed by internal crystallization h_f to the total depth H of the lava lake as a function of H. The dashed lines indicate the results for magma chambers, which lose heat by conduction into the overlying country rock [*Huppert and Worster* 1992].

DISCUSSION

The thermal history and petrogenesis of the lava lakes of Kilauea Iki, Makaopuhi and Alae on Hawaii have been documented in considerable detail [*Wright et al.*, 1976; *Wright and Okamura*, 1977; *Peck et al.*, 1977; *Peck*, 1978]. The results of our models cannot yet be expected to agree entirely with these studies, because there is a lack of data on the kinetic crystallization laws of minerals at small undercoolings in basaltic melts, and because there are a number of complexities in these lava lakes which have yet to be incorporated into a theoretical model. The lava lakes had quite complex filling histories, and the internal temperature and composition variations at the time a solid continuous crust first formed are uncertain. Vesiculation also played a significant role and, in the case of Makaopuhi lava lake, inhibited convection in the first few months of crust growth [*Wright and Okamura*, 1977]. In the Kilauea Iki lava lake, vesiculation appears to have been even more prominent [*Helz*, 1987] with convective processes being confined to a 30-m layer of differentiated melt

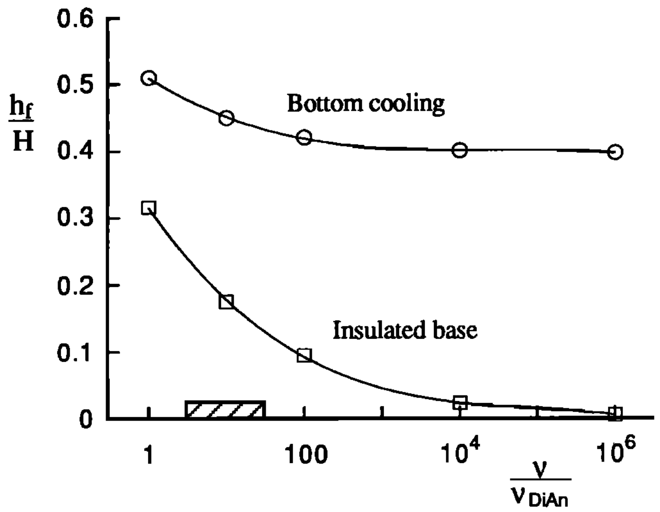

Fig. 9. Ratio of the final depth of the floor layer to the total depth of the lava lake, for a lake initially 100 m deep, showing how this ratio varies with the kinematic viscosity of the lava. There is little change in the depth of the floor layer with viscosity when heat is tranferred to the country rock, but there is a significant effect of viscosity when the bottom of the lake is insulated. The range of typical viscosities for basalt are sketched as a hatched region on the axis.

case, so that convection is still strong and may cause chemical effects not incorporated into the present model. On the other hand, when cooling to the country rock below is taken into account, the model predicts that the amount of solid formed at the bottom of the lake is little affected by the viscosity of the melt.

(cf. Figure 5 [*Wright et al.*, 1976]. The Alae lava lake degassed throughout most of its cooling history and the solid crust is highly vesicular, particularly in the top 6 m [*Peck*, 1978]. Despite these complexities and differences, our model does provide some new insights into the interpretation of data from Hawaiian lava lakes.

Temperature profiles through the growing crust of Hawaiian lava lakes all show good agreement with a purely conductive model [*Wright et al.*, 1976; *Peck et al.*, 1977]. There are, however, minor deviations from the simplest model which can be attributed to a variety of observed effects and plausible but uncertain variations of thermal properties of the growing crust. In the early stages of growth the surface temperature is greater than that of the atmosphere. Significant amounts of rainfall which seeped through the crust also contributed later to depressing temperatures in the uppermost parts of the crust to values lower than would be expected. The detailed numerical model of the Alae lava lake [*Peck et al.*, 1977] employed variations in thermal conductivity with temperature and vesicularity to obtain a best fit result to a conductive model. Our results show that such good fits do not demonstrate that convection was absent or unimportant in the interior. In the case of the Alae lava lake the temperature and vesicularity profiles in fact indicate that convection did not occur, probably as a consequence of degassing [*Peck*, 1978]. Our calculations show that models including convection will not be easily distinguishable from purely conductive models using data on crustal growth rates only.

Observations on the Makaopuhi lava lake support convective behavior in the interior and can be compared with the predictions of our model. During the first 8 to 9 m of crustal growth, over a period of about 7 months, the composition of the growing crust remained constant and olivine phenocrysts were retained [*Wright and Okamura*, 1977]. Temperature increases with depth in the lava below the boundary between melt and crust, which was defined by Wright and Okamura as the 1065°C isotherm at about 55% crystallinity. They inferred from the vesicular character of the crust in this period that vesiculation and rise of bubbles counteracted both convection and olivine settling. The lack of the internal chemical differentiation provides further support for the interpretation that convection was suppressed. Our model does not incorporate vesiculation. However, a volumetric fraction of bubbles of only 3×10^{-4} is sufficient to counteract the unstable density contrast associated with a temperature difference of 6°C. This calculation indicates that a modest upward gas flux can inhibit convection, as also suggested by *Davaille* [1991].

In Figure 10, we have plotted some measurements of the depth of crust in Makaopuhi lava lake [*Wright and Okamura*, 1977]. On the same figure we show the results of calculations of our model for a lava lake 84 m deep (the same as Makaopuhi). The parameter values used in the calculation were those of the Di-An system but with a viscosity 30 times greater, as appropriate for a basalt. There is approximate agreement between the theoretical results and the field data, although the two should not strictly be compared, since the parameter values of the Di-An system are not all the same as those of the basalt that constituted the Makaopuhi lava. There is a desperate need for more data on the thermal and physical properties of real lavas in order that more accurate theoretical predictions can be made.

After the crustal thickness in Makaopuhi had increased beyond 9 m, several observations indicate that convection began. Significant temperature fluctuations were observed in the temperature range 1130°C to 1140°C, with these isotherms being observed to move up and down. At this stage olivine phenocrysts were no longer present in the growing crust and had settled out. From 9 to 16 m depth the crust grew from an increasingly evolved magma. For example, the MgO content of the crust changed from about 8.3% to 6.4%. *Wright and Okamura* [1977] calculated that this differentiation required fractionation of approximately 30% crystals of olivine, plagioclase, and clinopyroxene. These compositional changes are smooth and linear with depth and are as predicted by our model, in which differentiation occurs by convection-driven interior crystallization and crystal separation. There is insufficient information from Makaopuhi to evaluate the associated temperature change, but it is likely to have been small as a consequence of the multiply saturated character of the Makaopuhi basalt in which large amounts of crystallization took place over a small temperature interval. *Wright et al.* [1976] note that samples of melt obtained at this stage were dense and lacked vesicles and proposed that thermal convection had begun.

The data from the historic lava lakes of Hawaii unfortunately do not record information on crustal growth when convection would be expected to have the most effect. In our model of a 100-m-deep lake the compositional changes due to convection-driven internal cooling and differentiation become increasingly important as the crust thickens (Figure 4*c*). In Makaopuhi data on crustal growth is only available to 16 m depth, and in the first 8 to 9 m vesiculation appears to have inhibited growth. Thus the data can only be compared with the first 10 to 15 m of the models, where only small amounts of internal cooling and differentiation are predicted. The internal differentiation of a prehistoric lava lake (100 m thick) has been documented by *Moore and Evans* [1967]. Their compositional profile indicates significant internal differentiation. The S-shaped profile can partly be explained by settling of olivine phenocrysts originally suspended in the lake [*Marsh*, 1988]. However, the evolved character of the lava at about 60 to 70 m height also requires significant internal cooling and differentiation. The most rapid changes in composition occur in the center as predicted.

An important issue in comparing the model with observations concerns the nature of the mushy layer in a real lava lake.

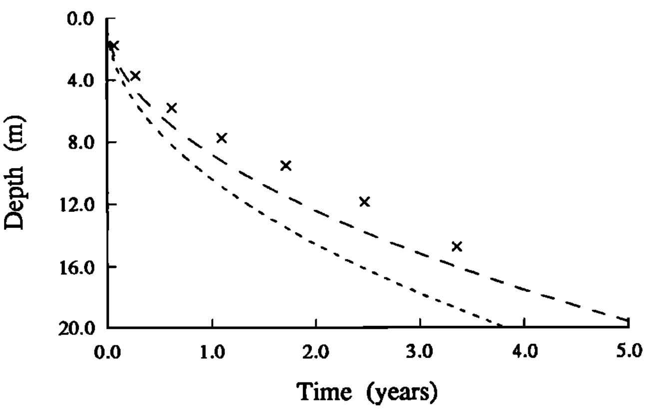

Fig. 10. The crosses show the measured depth of crust in the Makaopuhi lava lake as a function of time. The solid line shows the position of the mush–liquid interface; and the dashed line, the position of the eutectic interface according to calculations for a lava lake 84 m deep containing lava with the physical properties listed in Table 2 (apart from the kinematic viscosity ν, which was taken to be 30 times greater, as appropriate for basalts).

In Hawaiian lakes the crust-melt interface has been defined as the temperature interval over which there is a dramatic drop in strength and viscosity during drilling. In the three lakes studied this boundary is quite precisely located at 1065° to 1070°C and a crystal content of about 55% [*Wright et al.*, 1976]. At greater depths the lava is regarded by these authors as part of the interior melt. However, the criterion based on drilling behavior is not necessarily appropriate from a fluid dynamical point of view. For example, the Makaopuhi lava contains well over 20% crystal even at 1140°C, at the greatest depths measured below the crust, thus the crystal contents are still sufficiently high that the interval between 1065°C and 1040°C could still be an interconnected mushy zone from a fluid dynamical perspective. What is not clear from available data is the crystallinity at which it is appropriate to regard the system as a slurry (isolated suspended crystals) rather than a mush. Our treatment of the system as a mush is one end-member. A slurry model would produce broadly similar results, but the convection would be more vigorous for two reasons. First, the growth of dense crystals in the active thermal boundary layer contributes to the unstable density difference, typically increasing the effective coefficient of expansion by a factor of 10 in basalt. Second, the temperature difference would be determined by the temperature-dependent viscosity gradients in the slurry and should be associated with larger driving ΔT than used here [*Davaille*, 1991].

Finally, our study of the influence of viscosity implies that internal convection caused by convection-driven undercooling will be much less important in high-viscosity magma. The absence of layering in granite intrusions may thus be partly a consequence of kinetic effects as also suggested by *Brandeis and Jaupart* [1986] for different reasons.

CONCLUSIONS

Calculations have been made of the solidification history of a hypothetical lava lake of a simple diopside–anorthite melt. This model system serves to illustrate important physical processes that can occur in real lava lakes. In particular, it demonstrates the important coupling of convection and solidification, whereby thermal convection is driven solely by the kinetic undercooling of solidification and causes secondary crystallization of the melt in regions of the lake that are far from the cooled boundaries.

Reasonable estimates of the time for complete solidification of lava lakes have been made by employing appropriate thermal boundary conditions; the top surface of the lake was assumed to be at a fixed temperature, while the time-varying heat flux out of the bottom of the lake was computed by solving for the heat conduction into the country rock below the lake. It was found that of the solid that formed at the bottom of the lake, about half formed in response to the cooling through the top surface of the lake and the remainder grew directly in response to the cooling from below.

The viscosity of the lava has a significant effect on the history of its solidification. Lavas with large viscosity will undergo much less internal crystallization caused by convection driven by kinetic undercooling.

Comparisons of our model with observations of Hawaiian lava lakes indicate that the history of shallow lakes (e.g., Alae) and the early history of deep lakes (e.g., Makaopuhi) are controlled by degassing in which bubble ascent dominates convection. However, when degassing has diminished, the internal differentiation and temperature fluctuations are as observed.

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