THE DYNAMICS OF PARTICLE-LADEN FLUID ELEMENTS

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ABSTRACT This work presents a new framework for modelling the suspension of sediment within a turbulent boundary layer. It aims to provide a link between the 'bursting phenomenon', which consists of the abrupt ejection of fluid away from the boundary, and the process by which sediment is resuspended. We focus on the advective transport of sediment by coherent motions within the boundary layer and propose a phenomenological model to describe a discrete ejection of particle-laden fluid. As the fluid element rises, it expands due to the entrainment of ambient fluid. This reduces its vertical velocity and particle concentration. The particle concentration is also reduced by gravitational settling, because the heavy particles sediment out of the fluid element. The model accounts for the dynamics of these particle-laden fluid elements by considering the conservation of their mass, momentum and particle concentration. Results indicate that this framework may provide valuable insight to the suspension of sediment by the 'bursting' process.

1. Introduction

Many recent studies have identified three dimensional coherent motions within turbulent boundary layers and have indicated that such motions are responsible for the production of turbulent kinetic energy. Comprehensive reviews of the dynamics of these motions have been published by Grass *et al.* (1991), Robinson (1991) and Smith *et al.* (1991). The existence of these coherent motions was first identified by Kline *et al.* (1967) in an experimental study of turbulent flow over a smooth boundary. They identified streaks within the viscous sub-layer, with alternating narrow elongated zones of high and low velocity fluid. They linked this structure in the viscous layer to a bursting event, which consisted of a sudden ejection of fluid away from the boundary, followed by a compensating inrush of fluid towards the boundary. These ejection and inrush events form the quasi-periodic patterns of the coherent motions within the flow and will henceforth be termed ejection and sweep events. The sweep event transports high momentum fluid towards the boundary, whereas the ejection transports low momentum fluid into the body of the flow. Experimental studies have demonstrated that these coherent motions are responsible for the major contributions towards turbulent energy production and Reynolds stress momentum transport. Similar conclusions are deduced from the field studies of Gordon (1974) and Heathershaw (1974). They noted the intermittent nature of the transport of momentum through a boundary layer of geophysical dimensions, which is consistent with the laboratory observations made at much smaller scales.

The role of turbulence in the process of suspending and entraining particles from an erodible boundary has also received considerable study. Studies have sought to form a link between coherent motions within the turbulent boundary layer and the suspension process (Grass 1974, Jackson 1976, Sumer & Oguz 1978, Deigaard & Sumer 1981). Jackson (1976) reasoned that the bursting process is a plausible mechanism for the suspension of sediment because it provides a means for the maintenance of the vertical anisotropy of turbulence. The bursts impart an upward momentum flux on the particles which exceeds the downward flux from the return flow. Grass (1974) filmed the suspension process by towing a flat bed of sand through an otherwise quiescent fluid. He identified coherent flow structures within the boundary layer and calculated the velocities of the particles advected by these motions. This observation links sediment suspension with the ejection of fluid away from the boundary. Further experimental studies have been performed by Sumer & Oguz (1978) and Sumer & Deigaard (1981), who observed the motion of heavy particles near to the bottom boundary of a turbulent channel flow. These observations confirm the role of the bursting process as a mechanism for sediment suspension.

Soulsby *et al.* (1987) have demonstrated an important link between the bursting process and the suspension of sediment within boundary layers of geophysical scale. They made simultaneous measurements of the high frequency fluctuations of the concentration of sand, suspended by a tidal current, and the horizontal and vertical components of the water velocity, above a sandy bed of an estuary. They showed that large upward fluxes of sediment are associated with ejection events.

We present in figure 1 some data collected by Grass (1974) which gives typical vertical and horizontal velocities for sand particles, which are suspended within an ejection event. We note that these profiles have the correct velocity signature for an ejection event, namely the particles are moving upward and their horizontal velocity lags the mean flow. Further we note that the horizontal velocity of the ejection lags the mean flow by an approximately constant value and that the vertical velocity reaches a peak at a height of approximately 300 viscous length units above the lower boundary. Grass (1983) attributes this peak to a complex vortical interaction between the coherent structures. The dynamics of this interaction are incompletely understood.

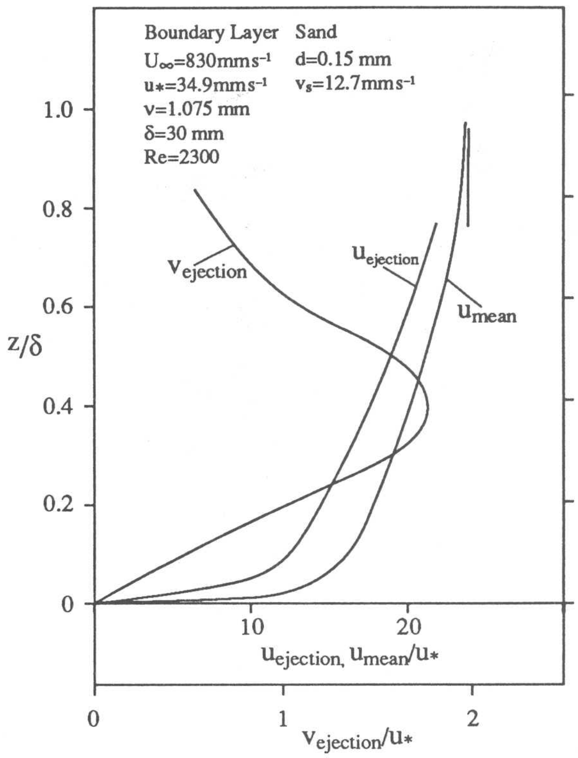

FIGURE 1 Streamwise and vertical velocities of sand particles suspended within ejection events from a flat-plate turbulent boundary layer. The velocities are non-dimensionalised with respect to the 'friction' velocity u_* and the height with respect to the boundary layer depth δ and the viscous length unit ν/u_* (reproduced from Grass 1983).

The present work is concerned with the fundamentals of sediment transport and focuses on the suspension of sediment by coherent structures within the boundary layer. We develop a phenomenological model of the advection of sediment by an erupting turbulent burst as it is ejected away from the the boundary region. This framework is similar to the approach of recent studies by Nielsen (1991) and Deigaard (1991) who also consider the advection of sediment by ejection events. However, here we account for the vertical transport of momentum and particles by the ejection.

The classical approach to the modelling of suspended sediment is to assume it follows gradient-diffusion behaviour. The non-isotropic characteristics of turbulence within the boundary layer are expressed through a diffusion coefficient which varies with distance from the boundary. Within the 'wall layer', in which the flow is turbulent but directly influenced by viscosity (Hinze 1975), the diffusion coefficient is often prescribed to vary linearly with distance from the boundary, reflecting the typical turbulent eddy dimension. Central to gradient-diffusion modelling is the principle of averaging over a number of random excursions which occur over scales significantly smaller than those over which the characteristics of the domain vary. However, within the turbulent boundary layer, we find these two scales are of a comparable magnitude. This is bourne out by the observations of motions which are coherent throughout the entire boundary layer. Hence transport by these large-scale advective events can not be neglected. Such a conclusion led Hinze (1975) to conclude that transport could not be described in a satisfactory way by means of gradient-diffusion alone.

The approach used here is to model the dynamics of one advective event and account for the resulting transport of particles and momentum. We develop a simple phenomenological model which aims to capture some of the fundamental physics of the suspension process. We demonstrate that within certain asymptotic regimes, this framework reproduces some 'well-known' results which are usually derived from a different set of assumptions. In the following section, we define the problem and introduce variables to describe a particle-laden fluid element. In §3, we present equations to describe the conservation of mass and momentum of the suspending phase and we introduce the entrainment assumption, following the work of Morton *et al.*

(1956). This assumption is vital to quantify how the fluid element mixes with the ambient fluid. These equations are integrated and the results presented in §4. We consider the particulate phase in §5 and study the implication for the average concentration profile. Finally, we present some conclusions in §6, noting that this analysis reproduces some of the experimental results of Grass (1974) for the vertical and horizontal velocities of an ejection event.

2. Particle-laden fluid elements

We consider the suspension of sediment under the simplest turbulent flow conditions, namely a steady mean flow $[u(z), 0, 0]$, driven by a steady pressure gradient, $\partial p/\partial x$. We have noted above that even within this simplest flow, coherent motions are observed within the boundary layer, the dynamics of which are incompletely understood. Introducing the Reynolds decomposition of the Navier-Stokes equation by writing the turbulent velocity components as (u', v', w') and neglecting viscous terms, we can write the time-averaged conservation of fluid momentum as

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\partial}{\partial z}\left(\overline{u'w'}\right). \qquad (1)$$

In this expression the Reynolds shear stress $\tau/\rho = -\overline{u'w'}$. Since the average pressure gradient driving the mean flow is constant and the shear stress vanishes at the free-surface $z = D$, we may integrate this equation to give

$$\tau = \rho u_*^2 (1 - z/D), \qquad (2)$$

where D is the flow depth, u_* is the friction velocity, defined by the Reynolds shear stress at the boundary ($\tau_0 = \rho u_*^2$) and the z-axis is taken away from the solid boundary. We note that for ejections $u' < 0$ and $w' > 0$ while for sweeps $u' > 0$ and $w' < 0$. Both make positive contributions to the shear stress.

We develop a model of a particle-laden fluid element and use it to describe the evolution of a turbulent ejection of fluid away from the boundary. The model describes the size and velocity of the event. As the fluid element is ejected away from the boundary, it entrains some of the ambient fluid through which it rises and it is advected downstream. As a result of the entrainment, the element not only mixes with ambient fluid, but also grows in size. Particles that are advected with the fluid element undergo gravitational settling and some fall out of it, whilst others, suspended in the ambient, are entrained into it. The framework developed below provides a simple way of accounting for this process and enables the calculation of the particle concentration within an ascending fluid element.

We describe the particle-laden fluid element using the variables shown in figure 2. We denote the radius of the spherical fluid element by b and its vertical velocity by w. The horizontal velocity of the element is denoted by v, while the mean horizontal velocity of the ambient is denoted by u. The entrainment velocity is denoted by u_e and represents the velocity of the ambient fluid across the surface of the expanding fluid element. We denote the volume concentration of particles within the fluid element and the ambient by C and C_a, respectively, both of which will be assumed to be small. The particles have a settling velocity given by V_s.

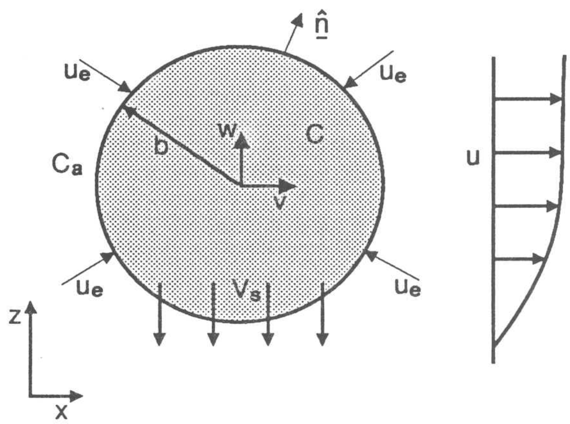

FIGURE 2 Variables used to describe the particle-laden fluid element (see §2).

The variables defined above are averaged over the timescale of eddies causing the entrainment into the fluid element. We treat the fluid element as an evolving sphere. Although at any instant the interface between the fluid element and the ambient is highly contorted by the presence of small-scale eddies causing mixing with the ambient, in an averaged sense the fluid element has a more regular shape. Three-dimensional shapes other than a sphere could be modelled by the inclusion of appropriate geometrical parameters, although this does not alter the inherent structure of the equations. We assume that the small-scale turbulence within the fluid element is sufficient to keep the particles well-mixed, so that we may use an average particle concentration. This framework for modelling the fluid elements follows the ideas first introduced by Morton *et al.* (1956).

3. Hydrodynamic model

We present equations to model the conservation of mass, horizontal and vertical momentum of the fluid element. We assume that the particle concentration is small ($C \ll 1$). This implies that the presence of the particles does not significantly affect the momentum of the fluid element. The particles are accounted for only in an equation expressing their conservation (see §5).

Central to the framework developed here is the entrainment assumption. This models the inflow to the fluid element as it ascends through and entrains ambient fluid. The model specifies the magnitude of the inflow velocity to be proportional to the vertical velocity of the element. The constant of proportionality is denoted by α and is termed the entrainment coefficient. Hence

$$\mathbf{u}_e = \alpha w \hat{\mathbf{n}}. \tag{3}$$

This assumption was first developed by Morton *et al.* (1956) and used to model the inflow to plumes and thermals. The use of the entrainment assumption postulates that the mixing with the ambient is driven by the vertical velocity.

3.1 Conservation of mass

As the fluid element ascends it entrains ambient fluid and so its volume increases. The principle of conservation of mass equates the rate of change of the volume with the rate of inflow over the boundary. Hence, in accordance with Morton *et al.* (1956), we find that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{4}{3}\pi b^3\right) = 4\pi\alpha b^2 w. \tag{4}$$

By noting that $\mathrm{d}z/\mathrm{d}t = w$, we conclude that

$$\frac{\mathrm{d}b}{\mathrm{d}z} = \alpha. \tag{5}$$

Hence the dimension of the ejections grows linearly with distance from the lower boundary. This result follows directly from the entrainment assumption and is independent of the actual velocity of the element. It is in accord with the observation of the typical eddy dimension within turbulent boundary layers which also is observed to increase linearly with distance from the boundary.

3.2 Conservation of vertical momentum

The fluid element is ascending through ambient which has no mean vertical velocity. Hence it does not entrain any net vertical momentum. Therefore the rate of change of the vertical momentum of the element is equal to a vertical forcing which we denote by F_v,

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{4}{3}\pi b^3 w\right) = F_v, \tag{6}$$

If the fluid element were impulsively hurled from the lower boundary, then there would be no vertical forcing. However observations of turbulent ejections suggest that they accelerate during some initial phase. Dynamically this is due to some complex vortical interaction and hence we require a vertical forcing, the magnitude of which may be expected to decrease as the element ascends. We discuss a possible form of this forcing in §3.6. Observations indicate that ejections may impinge upon the free-surface of the flow (Jackson 1976). However, interaction with the free-surface of the channel flow is not included in this model and we find that the element has some small vertical velocity as it reaches the surface. Observations of a vortex pair impinging on a free-surface indicate that the surface is deformed (Ohring & Lugt 1991). However we do not include this effect as little sediment is advected to this height above the boundary.

3.3 Conservation of horizontal momentum

The ambient fluid has net horizontal momentum, which the fluid element entrains as it ascends. The element is also acted upon by a drag force, which we denote by F_h, and which is the horizontal component of the force explicitly discussed in §3.6. We expect that sufficiently distant from the boundary, the rate of change of horizontal momentum is balanced by the entrainment of ambient momentum, rather than the drag force F_h. The equation expressing this conservation relation is given by

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{4}{3}\pi b^3 v\right) = 4\pi\alpha\rho b^2 w u + F_h. \tag{7}$$

3.4 Global conservation of mass

In order to preserve the global conservation of mass within the channel, the ascending ejection drives a vertical return flow. If we assume that the average areal separation on the lower boundary of ejection events, which are initiated at the same time, is A, then the return flow w_r is given by,

$$\left(A - \pi b^2\right) w_r = \pi b^2 w. \tag{8}$$

The area A is a statistical quantity and may be a function of the outer flow variables. This continuity expression assumes that the return flow velocity is constant over the entire area A. Also tacitly assumed is that the spatial frequency of the ejections is such that they do not overlap (i.e. $A > \pi b^2$ throughout the channel depth).

3.5 Shear stress

The Reynolds shear stress is the rate of vertical transfer of horizontal momentum by turbulent motion. Using the framework developed here, we calculate the shear stress contributed by an ejection. The ascending fluid element carries fluid moving with horizontal velocity $v(z)$. Conversely the return flow driven by the element carries fluid moving with velocity $u(z)$. Hence the shear stress due to one of these elements passing is given by,

$$\begin{aligned} \tau_{ejection}/\rho &= -\frac{1}{A}\left(\pi b^2 wv - \left(A - \pi b^2\right) w_r u\right), \\ &= \frac{\pi b^2 w}{A}(u - v). \end{aligned} \tag{9}$$

We wish to link the shear stress caused by ejections to the mean shear stress within the channel, which varies linearly with distance from the boundary. This necessarily involves some averaging over the magnitude of ejection events. Observations suggest that the burst and sweep events contribute most of the turbulent shear stress and that they occur for a fairly constant fraction of time. Soulsby (1983) finds that ejection events occur for 10-13% of the time at heights of 30cm and 140cm above the bed and that they contribute 55% of the shear stress. We assume that ejections contribute a fixed proportion of the shear stress during a fixed proportion of time and denote these fractions by β and f respectively. Then the average turbulent shear stress is given by

$$\tau_{average}/\rho = \frac{f}{\beta A} b^2 w (u - v). \tag{10}$$

3.6 Drag Force

As part of the discussion of coherent motions (§1), we noted that it is postulated that ejections are initiated by vortical motions associated with earlier ejection events. Such vortical motions interact with fluid near the boundary, causing the eruption of the boundary layer. This behaviour is found in the phenomenological studies of the interaction of an inviscid vortex with a laminar boundary layer (Dogalski & Walker 1984). Their study demonstrates that the boundary layer erupts within a finite time.

We assume that the force exerted on the erupting fluid element, the vertical and horizontal components of which we have denoted by F_v, F_h, may be modelled by a non-linear drag law involving the velocity field associated with an overpassing ejection, initiated upstream at an earlier time ($t = -T$). The force is linked to the velocity difference between the overpassing eruption and the ambient fluid. Thus we propose,

$$\mathbf{F} = 4\pi\rho C_D b^2 |\mathbf{U}_I| \mathbf{U}_I, \tag{11}$$

where C_D is a constant, representing a drag coefficient, b is the radius of the erupting ejection and $\mathbf{U}_I$ is the velocity field associated with the overpassing ejection. The velocity field $\mathbf{U}_I$ is treated as a vortex of radius $b(t+T)$, with a constant radius of the inner vortical core a, moving with the velocity of an ejection initiated at $t = -T$, relative to the ambient flow. Hence we find that

$$\mathbf{U_I} = \frac{a}{b(t+T)}\mathbf{u}'(t+T), \tag{12}$$

where $\mathbf{u}' = (u - v, 0, w)$. We emphasise that this expression for the force on an erupting fluid element represents the vortical interaction with other events and not the interaction with the ambient, which is included via the process of entrainment. As noted by Grass (1983), the actual form of this interaction is complex and not fully understood. However we demonstrate that this model permits the derivation of well-known asymptotic results within the constant stress layer.

4. Hydrodynamic results

The equations (2)–(12) form a system of equations which may be integrated to find the evolution of the dimension and velocity of the fluid element. The equations involve a number of physical constants for which we prescribe values from a preliminary study of experimental measurements. If the system of equations is non-dimensionalised with respect to the initial velocity and radius of the fluid element, we find there are seven non-dimensional entities in the equations (1)–(12). It is emphasised, though, that the inherent structure of the equations remains unchanged by the chosen values of the constants. Graphs displaying the results of a numerical integration of the equations are presented in figures 3–5. In these figures, the velocities are non-dimensionalised with respect to the initial vertical velocity and height is non-dimensionalised with respect to the flow depth. The values of the constants used for this integration are given in table 1. Many of these values have some statistical distribution and a preliminary study of field data suggests that many are functions of the friction velocity u_*. This observation is reasonable, since the magnitude of many of these parameters will be determined by the magnitude of the turbulent fluctuations, which in turn is reflected in the magnitude of u_*.

Entrainment coefficient	α	0.2
Friction velocity	u_*/w_0	1.03
Drag coefficient	$C_D a^2/b_0^2$	0.33
Flow depth	D/b_0	50
Proportion of stress due to ejections	β	0.5
Proportion of time with ejections occurring	f	0.1
Areal frequency	A/b_0^2	1000

TABLE 1 Parameter values used to produce figures 3–5

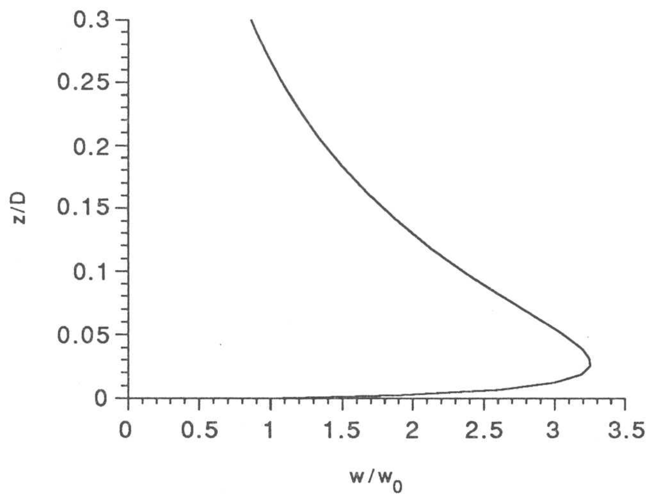

FIGURE 3 Vertical velocity profile of an ejection event.

We observe that the vertical velocity of the ejection increases to some peak value at around $z = b_0$ before decaying. In physical terms, the vertical forcing initially increases the momentum of the element by increasing its vertical velocity. However the ascending ejection

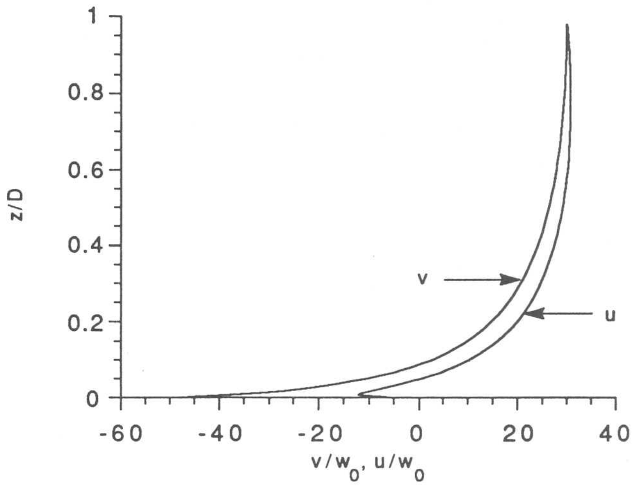

FIGURE 4 Horizontal velocity profile of an ejection event (v) and of the ambient (u).

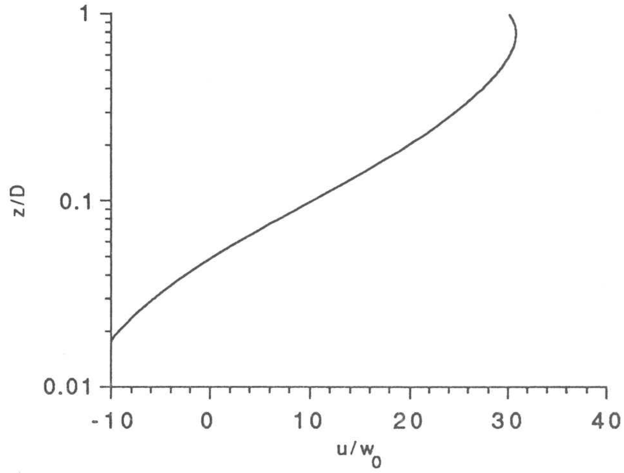

FIGURE 5 Horizontal velocity profile of the ambient shown on a logarithmic scale for distance from the boundary.

is also entraining fluid which has to be accelerated to the upward velocity. Far from the boundary, there is a balance between the acceleration of entrained fluid and the forcing and so the overall vertical velocity of the element decays. We find that the decay with height is proportional to z^{-2}, which is consistent with the experimental data presented by Grass (1983).

The horizontal velocity deficit of the ejection is approximately constant within a region close to the bottom boundary. Again this mirrors the experimental observations of Grass (1983). Furthermore we find there is a region in which the ambient velocity profile exhibits a logarithmic dependence on distance from the boundary. This reproduces the classical 'wall layer' result.

We find the following asymtotic results in the region, corresponding to the constant stress layer, $D \gg Z \gg b_0$

$$b \sim \alpha z, \tag{13}$$

$$w \sim u_* \sqrt{\frac{3C_D a^2 A\beta}{f\alpha^5}} \frac{1}{z^2}, \tag{14}$$

$$u - v \sim u_* \sqrt{\frac{A\beta\alpha}{3C_D a^2 f}}, \tag{15}$$

$$u \sim 4u_* \sqrt{\frac{A\beta\alpha}{3C_D a^2 f}} \log z. \tag{16}$$

These asymptotic results confirm the graphical observations made above. Furthermore within the framework of this analysis, the eddy viscosity is given by $b^3 w/A \sim z u_* \sqrt{3\alpha C_D a^2/fA}$; it varies linearly with distance from the boundary which reproduces the traditional diffusion result.

5. Particle concentration

We derive the following particle conservation equation for the fluid element

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{4}{3}\pi b^3 C\right) = 4\pi\alpha b^2 w C_a - \pi V_s b^2 C. \tag{17}$$

This equates the rate of change of the number of particles within the fluid element to the number entrained from the ambient minus the number which settle out. We assume that the turbulence within the fluid element is sufficient to keep the particles well-mixed and that they sediment with their settling velocity once they have left the element. Bonnecaze *et al.* (1993) present a similar approach to modelling the particle concentration within a turbulent flow, although they applied the technique to a different problem.

The fluid element is rising through an ambient in which particles are suspended. Therefore we include the entrainment of these particles into the element. We apply a steady-state principle of the global conservation of particles (cf. §3.4). We introduce an area A over which just one ejection event occurs and invoke the steady-state balance of the sedimentation of particles within the ambient with those brought up by the sporadic ejections, with a typical period T. Hence we find that

$$\int_z^{z+V_s T} A C_a \,\mathrm{d}z = \tfrac{4}{3}\pi b^3 C. \tag{18}$$

This closes the system of equations for the concentrations of particles in both the ambient and fluid element. Since we assume that ejections only occur for a constant fraction of any interval, we may calculate the average concentration profile. This is a temporal average of the concentration within the ambient and the fluid element as it ascends. Also we take a spatial average, because there is only one ejection event per area A, at any time. Denoting the average ambient concentration of particles by $\overline{C}_a$, we find that

$$\overline{C}_{average} = (1-f)\,\overline{C}_a + f\left(\left(1 - \pi b^2/A\right)\overline{C}_a + \pi b^2 C/A\right), \tag{19}$$

where $\overline{C}_{average}$ is the average particle concentration in the flow. We present some typical concentration profiles in figure 6, for varying values of the settling velocity of the particles. We note that the value of the parameter V_s/w_0 determines the shape of the concentration profile. This is equivalent to the role of the parameter V_s/u_*, which is used in the classical diffusion model of suspended sediment. It expresses the ratio of the settling velocity of the particles to the magnitude of turbulent velocity fluctuations. We note the concentration profiles are convex upwards.

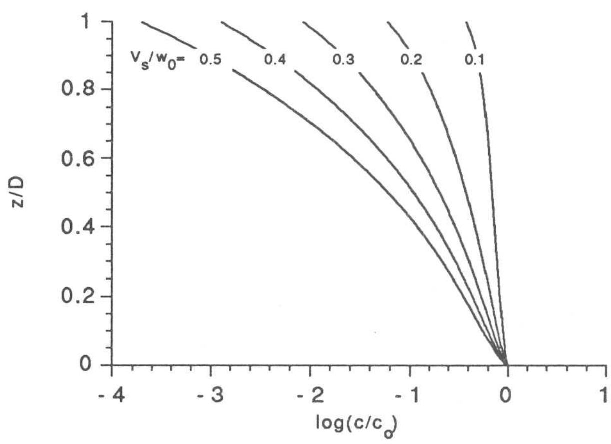

FIGURE 6 Average concentration profiles, non dimensionalised with respect to the concentration at the bed, for various ratios of the settling velocity to the initial vertical velocity of the ejection (V_s/w_0).

We may also perform asymptotic analysis to find an expression for the concentrations within the fluid element and within the ambient. In the regime $z \gg V_s T$, we find that

$$C_a = \frac{\frac{4}{3}\pi b^3}{AV_sT}C. \tag{20}$$

Furthermore, within the constant stress layer $D \gg z \gg b_0$, we find that

$$C \sim \frac{1}{\alpha^3 z^3}\exp\left(-\frac{3V_s}{8u_*}\sqrt{\frac{f\alpha^3}{3C_d a^2 A\beta}}z^2 + \frac{4\pi\alpha^3}{3AV_sT}z^3\right). \tag{21}$$

6. Conclusions

We have developed a model of sediment suspension that is based on the advective transport of particles by large-scale coherent events within turbulent boundary layers. This model accounts for the vertical transport of both particles and momentum by studying the dynamics of particle-laden fluid elements. We have demonstrated that by using this analysis we may reproduce some of the experimental observations made by Grass (1983). Furthermore, within the constant stress layer, we recover the logarithmic profile for the ambient velocity.

Whilst the model develops a framework for the analysis, there are many additional features of the flow which are not included at present. We have considered only the ejection events and have included various parameters, the values of which we have determined by only a preliminary study of data. Also we have included a forcing expression (§3.6) which requires further justification.

This framework could be applied to the modelling of the suspension of sediment for non-planar beds with waves and currents. For these scenarios, the initiation and scale of the coherent events which advect the particles away from the boundary are different, but it may be possible to apply the principles developed here to model sediment transport.

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