5 A Model of an Impinging Jet on a Granular Bed, with Application to Turbulent, Event-driven Bedload Transport

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ABSTRACT

A model of the impingement of a jet onto a granular bed is developed to calculate the volume of mobilized particles, as a function of characteristics of the jet and the grains. This model comprises a phenomenological description of bedload transport, which aims to form a new, quantitative link between coherent turbulent motions within turbulent boundary layers and the movement of sediment. The turbulent bursting phenomenon has for some time been identified as a dynamically important means of transporting momentum across the boundary layer and generating turbulent kinetic energy. It comprises two predominant types of motion, ejections and sweeps, both of which yield positive contributions to the Reynolds shear stress. While the abrupt ejection of fluid away from the boundary is a significant means for the suspension of particles, this paper considers an idealization of the impact of downward and fast moving sweep events on an erodible granular bed in terms of an impinging jet. A framework of analysis is proposed to model the way in which the impingement of the flow leads to the mobilization of particles from the boundary, the volume of entrained particles being specified as a function of the characteristics of the hydrodynamic event and the particles. By invoking a balance between the volume of entrained particles and the volume deposited between events, the bedload transport under equilibrium conditions is obtained. The predictions of this model are in accord with experimental observations as well as other derivations which adopt an entirely different approach. Furthermore, the quantitative calculations elucidated here lead to a mathematical relationship between the rate of bedload transport and the hydrodynamic characteristics of the flow which agree with previous empirical formulations.

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INTRODUCTION

The experimental studies of Kline *et al.* (1967) and Grass (1971) were among the first to identify unsteady, yet coherent, three-dimensional motions within turbulent boundary layers. They discovered that various flow patterns which exhibit irregular, but repetitive structures are found within turbulent boundary layers and that these patterns are coherent in both a temporal and spatial sense. These experimental studies identified predominant features of the flow: an abrupt ejection of fluid away from the boundary with a compensating inrush (sweep) towards the boundary and low-speed streaks within the viscous sublayer (see review by Smith, this volume). Collectively these are known as the turbulent bursting phenomenon. Observations of such features are common to all turbulent boundary layers, although the low-speed streaks are less pronounced for flows over rough boundaries for which the roughness elements inhibit the establishment of a viscous sublayer. However, Grass *et al.* (1991), Grass & Mansour-Tehrani (this volume) and Defina (this volume) find low-speed streaks above the grain tops. In addition to these laboratory studies, the bursting phenomenon has been noted in direct numerical simulations of bounded turbulent flow (Moin & Kim, 1985) and in field studies, where the boundary layer is of geophysical dimensions (Gordon, 1974; Heathershaw, 1974).

It is postulated that the origin of these coherent turbulent motions is due to vortex deformation within the flow. While a precise dynamical description of this evolution and interaction has not yet been formulated, some recent studies (Robinson, 1991; Smith *et al.,* 1991; Smith, this volume) have sought to draw upon the conclusions of various illustrative studies of vortex motion to suggest a plausible mechanism for the generation and maintenance of turbulence. Central to these mechanisms is the bursting phenomenon because it is responsible for the production of turbulent kinetic energy. Furthermore, the ejection and inrush events (which will henceforth be referred to as ejections and sweeps) make substantial contributions to the vertical rate of transfer of horizontal momentum by turbulent motions, known as the Reynolds stress, and are hence essential momentum transport mechanisms across the boundary layer.

The role of turbulent motions in the process of transporting sedimentary particles has been the focus of considerable discussion. Studies have sought to form a link between the coherent turbulent motions within the turbulent boundary layer and the transport of sediment (Jackson, 1976; Sumer & Deigaard, 1981). At a most basic level, it seems apparent that the interactions of vortex-like structures within the flow must have a dominant influence on the dynamics of sediment transport. The ejection events transport particles away from the boundary, while sweep events mobilize particles which are in repose on the boundary. Whereas recent papers have examined the way in which ejection events may be involved in the process of sediment suspension (Hogg *et al.*, 1994), the current work examines the role of sweeps in the mobilization of particles at the bed.

Grass (1971) noted that not only do sweep events play an important role in the general mechanism of the vertical transport of momentum, but they were capable of mobilizing sand particles in repose on the smooth lower boundary of a laboratory flume and transporting them over considerable distances downstream. He found that the longitudinal scales of these events were much greater than the lateral scale. Drake *et al.* (1988) concluded that when a sweep impinged on a bed, particles were entrained at a rate many times the average and were transported downstream faster than the average flow velocity. They found that the sweep-transport events accounted for 70% of the total transport of sediment, even though they occurred for only 9% of the time. This reaffirms the considerable care which is needed when assessing sediment transport strictly from mean stresses rather than from instantaneous stresses, at least at low rates of transport. A similar conclusion may be drawn from the field study of Heathershaw & Thorne (1985). They used a novel acoustic measurement to link turbulent velocity fluctuations with the movement of sediment and found that the sweep events led to appreciable sediment transport. They also noted that the rarely occurring outward interaction events, which move faster than the mean flow and away from the boundary, mobilized more sediment per event than any other type of turbulent motion. Outward interactions occur relatively infrequently, however, rendering the sweep event as the most significant means for moving bedload particles. Heathershaw & Thorne (1985) also demonstrated that sediment movement exhibits a better correlation with streamwise velocity than with kinematic Reynolds stress. This observation suggests that bedload transport is not driven solely by bed shear stress, but additionally by forces arising from pressure fluctuations within the bed. In the studies of Drake *et al.* (1988) and Heathershaw & Thorne (1985), the beds were comprised of fine gravels, which were sufficiently large that they were unlikely to be entrained into suspension under most flow conditions, but rather were transported in the bedload, predominantly driven by the intermittent sweep events.

In this paper a framework for modelling the impingement of a jet on a granular bed is presented and possible applications to bedload transport are discussed. It is proposed that incoming sweep events may be treated as impinging jets and the model is used to study their interaction with an erodible bed of grains. The volume of particles mobilized by a single event is quantified as a function of the characteristics of the jet and the particles. The next section of this paper discusses experimental measurements of the rate of bedload transport and reviews the concept of bulk erosion, first introduced by Fernandez-Luque (1974), which suggests a new approach to modelling the bedload transport. The mathematical model of the impinging jet is described, separating it into essentially three distinct stages. This model permits evaluation of the rate of transport in equilibrium conditions.

BEDLOAD TRANSPORT

The motion of particles within the bedload may be distinguished from those in suspension by the following simple mechanistic definition. The weight of the particles within the bedload is never fully supported by the fluid motion, whereas within the suspended load this balance is possible. Hence, within the bedload, particles roll, slide and saltate along the underlying bed. With sufficiently energetic flows, which mobilize highly concentrated layers of particles close to the bed, it is possible for the weight of the particles to be supported by inter-granular interactions. This regime is termed 'sheet-flow', because the particles move over each other in highly concentrated layers.

There have been a number of experimental studies of steady-state bedload transport for a range of flow conditions and types of sediment. These experiments, conducted in laboratory flumes, have measured the flux of bedload transport, q_{bl}, which is defined by

$$q_{bl} = \int_0^D C(y)u(y)\mathrm{d}y \tag{5.1}$$

where $C(y)$ is the concentration of particles in the bedload, $u(y)$ is their horizontal velocity, y is the distance normal to the bed and D is the flow depth. This flux can be non-dimensionalized with respect to $\sqrt{(s-1)gd^3}$, where s, g, d are the ratio of the solid to fluid densities, gravitational acceleration and the diameter of the particles, respectively. The bedload transport flux is typically measured as a function of the Shields parameter (θ) based on the tractive stress exerted by the flow on the bed (skin friction). This parameter expresses the ratio of the drag exerted by the flow on a particle to the weight of the particle. For a turbulent flow in which the magnitude of the skin friction (σ_b) may be characterized by a friction velocity ($\sigma_b = \rho u_*^2$), the Shields parameter is given by

$$\theta = \frac{u_*^2}{(s-1)gd} \tag{5.2}$$

Shields (1936) studied the initiation of particle motion and elucidated the critical value of this ratio (θ_c) which must be exceeded before particle motion is initiated from a bed of non-cohesive grains. Shields demonstrated how this parameter varied with particle Reynolds number based on the particle diameter and found that for water flows and particle Reynolds numbers in excess of unity, $\theta_c \approx 0.05$. When the Shields parameter just exceeds the critical value for the initiation of motion, the particles roll, slide and saltate along the underlying bed. At considerably higher values of the Shields parameter ($\theta > 1$), a regime of sheet-flow is entered in which the weight of the particles is balanced by gradients of the dispersive pressures arising from particle interactions.

Figure 5.1 shows the variation of the non-dimensional rate of bedload transport against the Shields parameter. The data covers over two orders of magnitude of the Shields parameter and exhibits a consistent trend. Meyer-Peter & Müller (1948) suggested the following empirical relationship between the rate of transport and the Shields parameter

$$q_{bl} / \sqrt{(s-1)gd^3} = 8\left(\theta - \theta_c\right)^{3/2} \tag{5.3}$$

There have been a number of subsequent empirical and semi-empirical models of bedload transport which exhibit the same fundamental dependence of the rate of bedload transport with $\theta^{3/2}$. For instance, Nielsen (1992) suggests that

$$q_{bl} / \sqrt{(s-1)gd^3} = 12\sqrt{\theta}\left(\theta - \theta_c\right) \tag{5.4}$$

The latter formula also matches the data collected at high values of the Shields parameter. In Figure 5.1, both of the relationships given by Equations 5.3 and 5.4 are shown.

Bagnold (1954) studied the normal and tangential stresses developed within a sheared suspension of particles. By repeating the experiment with a pure fluid and comparing results, he was able to deduce empirical relations for the inter-granular forces. He noted that the total shear stress of a suspension is a combination of stresses arising from the fluid and particulate phases. This led Bagnold (1954) to postulate that for an equilibrium state, the fluid shear stress at a vertically uniform bed must be reduced to the threshold value for motion; he argued that if the fluid shear stress exceeds the threshold, then particles would continue to be entrained from the bed. The application of this condition, together with an expression of a dynamic friction law which links the shear and normal stresses, implies that the concentration of particles within the bedload may be deduced without explicitly specifying the volume of entrained and deposited particles. This approach has permitted the prediction that the steady-state bedload transport varies with $\theta^{3/2}$ (Engelund & Fredsøe, 1976), which concurs with experimental studies. However, it has not been possible to extend this procedure to study non-equilibrium transport. In the course of this paper, a new framework for the analysis of bedload motion is presented which is driven by the transport arising from individual coherent turbulent motions.

TOWARDS A NEW MODEL OF BEDLOAD TRANSPORT

Within a turbulent boundary layer flow there are velocity fluctuations which lead to fluctuations of the local pressure field and influence the motion of particles in the bedload. Noting this effect led Fernandez-Luque (1974) to introduce a concept of bulk erosion, a local phenomenon where large, instantaneous and localized pressure gradients within the turbulent boundary layer can lead to entrainment of particles from the boundary. From consideration of the forces exerted on the granular bed by

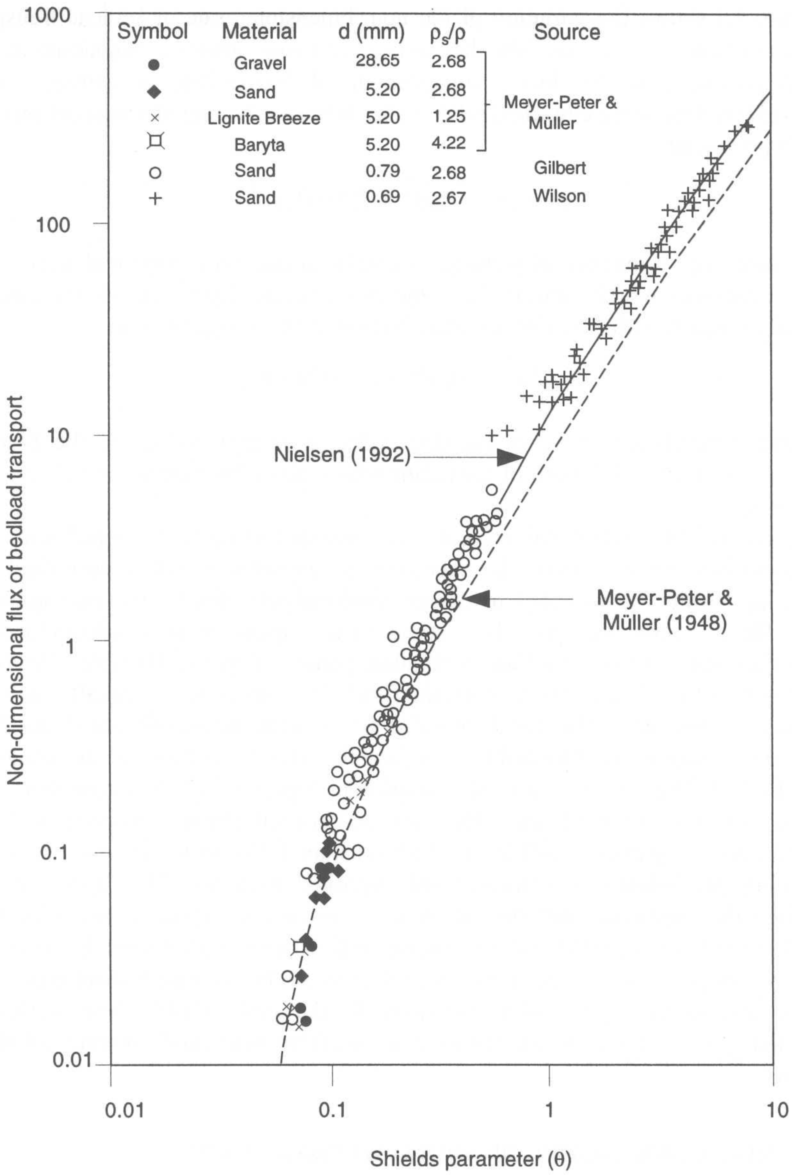

Figure 5.1. The steady-state rate of bedload transport. The dimensionless rate of transport ($q_{bl}/\sqrt{(s-1)gd^3}$) is plotted against the Shields parameter (θ), based on skin friction. The curves shown correspond to the equations proposed by Meyer-Peter & Müller (1948) ($q_{bl}/\sqrt{(s-1)gd^3} = 8(\theta-\theta_c)^{3/2}$) and Nielsen (1992) ($q_{bl}/\sqrt{(s-1)gd^3} = 12\sqrt{\theta}(\theta-\theta_c)$). Reproduced from Nielsen, 1992

the individual events, Fernandez-Luque (1974) defined a local and instantaneous bulk erosion boundary to represent the volume of grains which could be mobilized by a single event.

As noted above, recent research has indicated the existence and importance of coherent motions within turbulent boundary layers. Of these motions, the violent ejection of fluid away from the bed and the compensating sweep towards the boundary are thought to be of the most dynamical significance for mobilizing sediment. While ejection events advect particles far away from the boundary, sweep events impinge upon the bed. They lead to pressure fluctuations within the boundary layer and exert a shear stress on the surface of the bed. They are thus suitable candidates to initiate bulk erosion as envisaged by Fernandez-Luque (1974). Motivated by the observations of bedload transport driven by sweep events and armed with the concept of bulk erosion, the current paper suggests a new framework for modelling the mobilization of sediment particles by turbulent events which seeks to calculate the mass of particles mobilized by events of a given magnitude. Such a calculation yields an effective pick-up function for the bedload transport (*cf.* Nielsen, 1992) which in this study is linked to the impinging events.

This paper proposes four stages of a recipe for modelling turbulent event-driven bedload transport, based upon the study of the impingement of a jet on a granular bed (see below, Figure 5.2). The basic framework of this approach is now discussed and in subsequent sections details are presented of the mathematical modelling. The model acts as a basis from which further studies may elucidate more clearly the nature of the sweep events and their interaction with erodible granular beds.

An incoming, fast-moving sweep event may be treated as a jet flow impinging on a granular bed and locally this flow will be similar to a stagnation point flow. This impingement creates a pressure and shear stress distribution along the surface of the bed, the magnitudes of which are determined by the hydrodynamic characteristics of the jet. Within the bed the effects of the surface shear stress are rapidly attenuated, and so pressure gradients are the main forces responsible for mobilizing the sediment (as demonstrated below). A stability criterion can be defined which gives a minimum value of the ratio of the driving to resisting forces on a particle and allows calculation of a zone of erosion. This permits determination of the volume of particles mobilized by the impinging event. The rate of bedload transport can then be calculated by invoking a simple balance between the volume of particles entrained by the event and those deposited between events.

It should be emphasized that this analysis is relevant only to the parameter range $\theta_c < \theta < 1$; the flow is sufficiently energetic to mobilize particles, but not to launch them into suspension or initiate a sheet-flow layer. Furthermore, the time-scale of the inertial response of the particle must be assumed to be much less than the duration of the event. Smaller particles (sands and fine gravel) are rapidly accelerated by the flow, whereas larger particles (gravels) have a much longer inertial time-scale and are insensitive to the turbulent fluctuations. The transport of

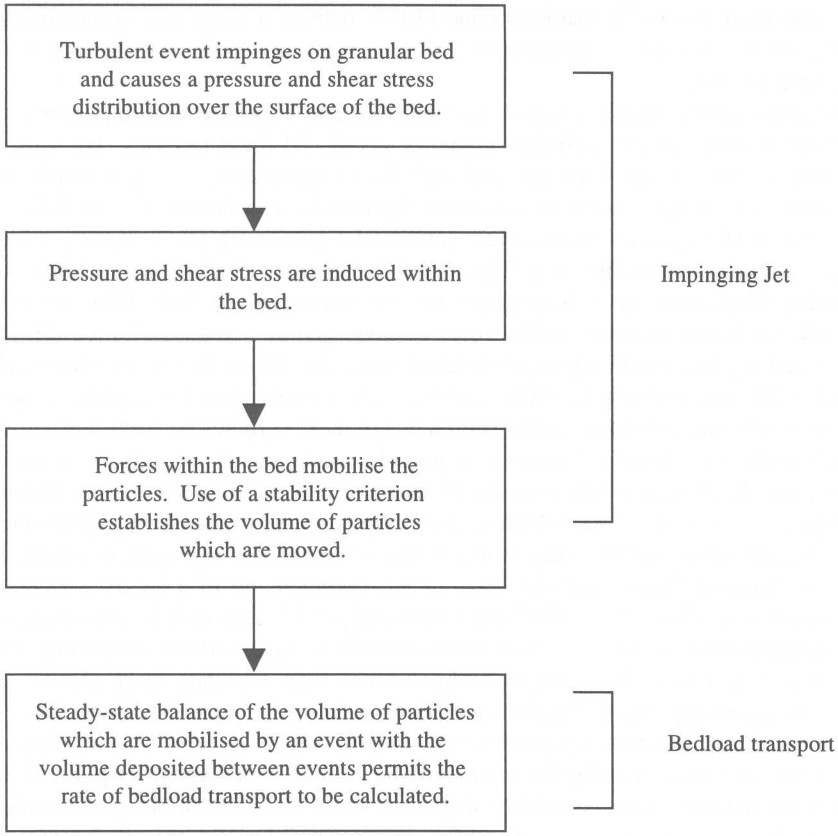

Figure 5.2. The four stages in the mathematical model of a jet impinging on a granular bed and its application to bedload transport

these larger particles is therefore poorly described by this model because they are incapable of responding to the fluctuating velocity field on a sufficiently rapid time-scale.

MATHEMATICAL MODEL OF THE IMPINGING JET FLOW AND BULK EROSION BOUNDARY

This paper considers the impact of an axisymmetric jet on an initially horizontal bed of cohesionless particles and calculates the erosion boundary, the concept of which was described above. The incoming jet is considered to be axisymmetric, because this is the simplest model of a three-dimensional jet flow. Additionally the axis of the jet is assumed to be normal to the granular bed, which is in accord with the viscous stagnation-point flow considered by Cleaver & Yates (1975) as a simple model of a sweep event. Furthermore, Raudkivi (1990) suggests that the angle of impingement has negligible influence upon the depth of erosion, although the erosion profiles become increasing asymmetric as the angle departs from the vertical.

IMPINGING JET FLOW

The impinging jet flow leads to distributions of shear stress and pressure on the surface of the granular bed. Beltaos & Rajaratnam (1974) and Rajaratnam (1976) performed experimental studies of jets impinging on a rigid plane, the configuration of which are shown in Figure 5.3. The source of the jet had velocity U_0, diameter a and was of a height H above the plane. The steady impingement of a jet onto an erodible bed exhibits substantially different behaviour from the impingement on a rigid boundary because the modification of the bed topography will influence the flow. However, the current model applies these experimental results for the rigid boundary, because, the jets considered here are active for only a short duration. Beltaos & Rajaratnam (1974) find experimentally that the pressure at the stagnation point is given by

$$p_0 = 25\rho U_0^2 / (H/a)^2 \tag{5.5}$$

and the surface pressure distribution is given by

$$p_s(r)/p_0 = \exp\left(-114(r/H)^2\right) \tag{5.6}$$

where r is the radial distance from the stagnation point. Furthermore, they find experimentally that the maximum shear stress exerted on the boundary is given by

$$\sigma_0 = 0.16\rho U_0{}^2 / (H/a)^2 \tag{5.7}$$

and note that far from the stagnation point, the shear stress decreases with the inverse-square of the radial distance. Hence the following equation can be proposed to describe the shear stress distribution:

$$\sigma_s(r)/\sigma_0 = 0.033\frac{\left[1-\exp\left(-114(r/H)^2\right)\left(1+114(r/H)^2+11.09(r/H)^3\right)\right]}{(r/H)^2} \tag{5.8}$$

This relationship is a slight amendment of that presented by Beltaos & Rajaratnam (1974) to fit the experimental data and to account for the far-field relationship.

The use of the experimental data of Beltaos & Rajaratnam (1974) fixes the value of the ratio of the maximum stagnation pressure (p_0) to the maximum shear stress (σ_0), given by $p_0/\sigma_0 = 150$. This value is applicable to an impinging jet but may not be appropriate for an impinging turbulent event. Johansson *et al.* (1987) studied the impingement of turbulent events on a smooth boundary and found that this

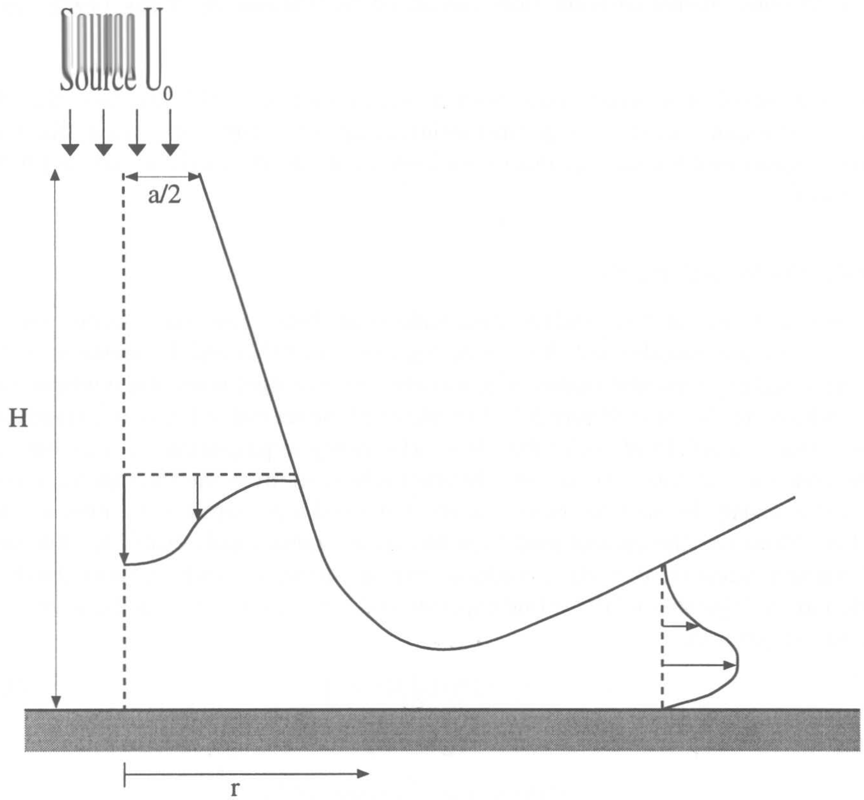

Figure 5.3. The radial wall jet arising from the impingement of a cylindrical jet

ratio is O(10). Nevertheless, the remaining stages of this analysis may be followed irrespective of the actual magnitude of p_0 / σ_0. Furthermore, to a first approximation, it is shown below that the functional form of the relationship between the volume of mobilized particles and the hydrodynamic characteristics of the impinging event is independent of this ratio.

FLOW WITHIN THE BED

Flow within the bed is modelled in the present study by the Brinkman equation for filter flow through a porous media (Wiegel, 1980). This is a modification of Darcy's Law, which permits the modelling of flow when the porous region is bordered by a pure-fluid region (Koplik *et al.*, 1983). The bed is assumed to be semi-infinite and statistically homogeneous. The difference between the fluid pressure and the hydrostatic pressure is denoted by *p*, the permeability of the porous bed by Λ and the dynamic viscosity of the fluid by μ. Hence the equations which govern the flow within the bed may be given by:

$$\mu\nabla^2\boldsymbol{u} - \frac{\mu}{\Lambda}\boldsymbol{u} = \nabla p \tag{5.9}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{5.10}$$

$$p \to 0,\ \boldsymbol{u} \to 0, \quad y \to -\infty \tag{5.11}$$

$$p = p_s(r), \quad \sigma = \sigma_s(r), \quad v = 0, \quad y = 0 \tag{5.12}$$

where $\boldsymbol{u} = (u, v)$ are the radial and vertical velocity components. The permeability of the bed is related to the grain size of which it is composed. Bear (1972) suggests an empirical relation for the permeability, measured in m^2

$$\Lambda = 0.617 \quad 10^{-3}\, d^2 \tag{5.13}$$

The following non-dimensional variables are introduced for the vertical and radial co-ordinates and the pressure and velocity fields

$$Q = p / p_0, \quad \boldsymbol{U} = \boldsymbol{u}\mu H / \Lambda p_0, \quad \xi = r / H, \quad \eta = y / H. \tag{5.14}$$

Noting that the pressure field satisfies Laplace's equation and providing that the surface pressure distribution is localized to the point of bed-impingement, the axisymmetric pressure distribution within the bed may be derived, via Fourier-Bessel transforms (see Appendix), as

$$Q(\xi, \eta) = \int_0^\infty \frac{k}{228} \exp\left(k\eta - k^2/456\right) J_0(k\xi)\, \mathrm{d}k \tag{5.15}$$

Isobars of this distribution are shown in Figure 5.4. Since the non-dimensional permeability $\lambda^{-2} \equiv \Lambda / H^2 << 1$, there is an 'effective' boundary layer within the flow through the porous media, adjacent to $y = 0$, within which viscous dissipation is non-negligible. Outside of this layer, Darcy stresses dominate and the velocity field is linearly related to the gradient of the pressure field (i.e. the first term of Equation 5.9 is negligible compared to the second). Within this boundary layer, variations in the vertical direction are more significant than those in the horizontal direction and so an approximate velocity field which satisfies the boundary conditions is given by

$$U = -\frac{\partial Q}{\partial \xi} + \left(\sigma_s(\xi)\lambda / p_0 + \frac{1}{\lambda} \frac{\partial^2 Q}{\partial \xi \partial \eta}\bigg|_{\eta=0} \right) \exp(\eta\lambda) \tag{5.16}$$

$$V = -\frac{\partial Q}{\partial \eta} + \frac{\partial Q}{\partial \eta}\bigg|_{\eta=0} \exp(\eta\lambda) \tag{5.17}$$

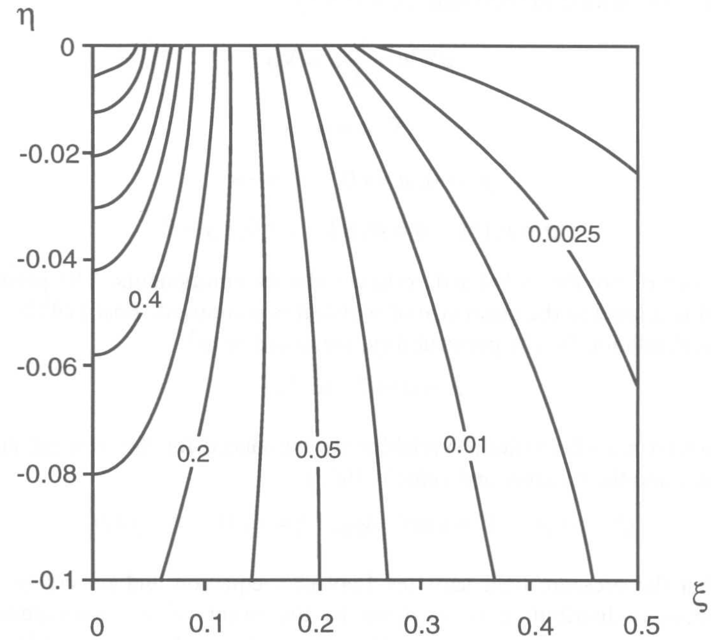

Figure 5.4. Isobars of $Q(\xi,\eta)$. The contours are shown for Q = 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.025, 0.01, 0.005, 0.0025, 0.001

Hence, the dimensionless force per unit volume acting on the particles, vertically averaged over a particle diameter, consists of radial and vertical components, given by

$$F_r = -\frac{\partial Q}{\partial \xi} + \frac{H}{d}\exp(\lambda\eta)\left(\sigma_s(\xi)/p_0 + \frac{1}{\lambda^2}\frac{\partial^2 Q}{\partial \xi \partial \eta}\bigg|_{\eta=0}\right) \tag{5.18}$$

$$F_y = -\frac{\partial Q}{\partial \eta} + \frac{H}{d}\exp(\lambda\eta)\frac{1}{\lambda}\frac{\partial Q}{\partial \eta}\bigg|_{\eta=0} \tag{5.19}$$

The extent of the boundary layer is $O(1/\lambda)$ which is also the extent of the region through which the influence of the surface stress is transmitted. It may be concluded, therefore, that the effect of the shear stress is absorbed by the granular bed over a relatively short, vertical distance (of the order of a grain diameter), whereas the pressure distribution is transmitted over a much greater length-scale. Similar conclusions are drawn by Fernandez-Luque (1974) and Wiegel (1980).

STABILITY CRITERION AND ZONE OF EROSION

This section considers the forces exerted on the particles which comprise the granular bed and invokes a stability condition to determine those which may be mobilized by the impingement of the incoming jet. This permits the definition of a zone of erosion and quantifies the mobilized particles as a function of the characteristics of the jet.

The forces acting on an individual particle in repose on a planar bed inclined at an angle β to the horizontal are considered; the forces which drive the motion in the radial and vertical direction are denoted by F_r and F_y respectively, while the weight of the particle is given by W. Motion is resisted by a frictional force which arises from the contact of the surface of the grains. The static angle of repose is denoted by α and is the angle at which the bed must be inclined before the grains start to move in the absence of any overlying fluid flow (for spherical, well-sorted sand grains, the angle of static repose is approximately given by $\alpha = 32°$). The configuration of the problem is shown in Figure 5.5.

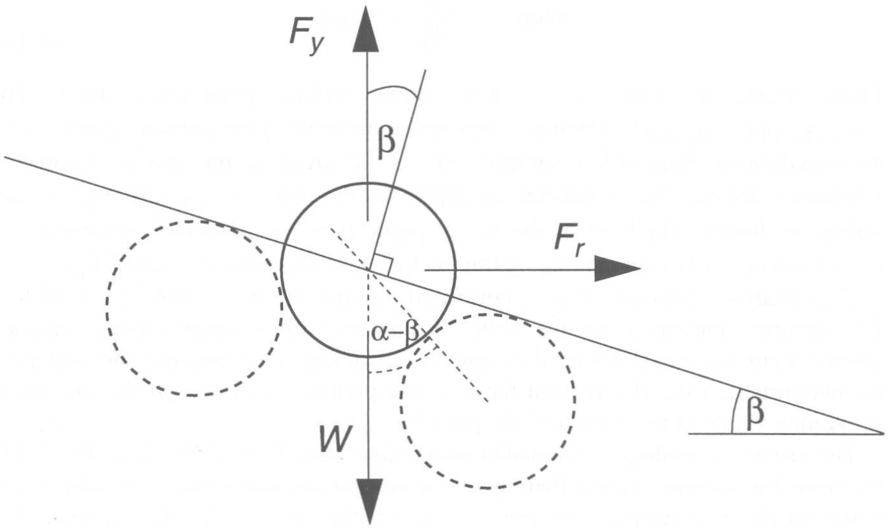

Figure 5.5. The forces acting on a particle in repose on a sloping bed. The horizontal and vertical components of the driving force are denoted F_r and F_y respectively, whilst the weight of the particle and the angle of static repose are given by W and $\tan\alpha$

An individual grain will start to move if the turning moment about its point of contact with a neighbouring grain exceeds the turning moment due to the weight of the particle. This condition is expressed by

$$F_r \cos(\alpha - \beta) + F_y \sin(\alpha - \beta) - W \sin(\alpha - \beta) > 0 \qquad (5.20)$$

Hence, motion occurs if

$$\frac{F_r}{-F_y + W} > \tan(\alpha - \beta) \tag{5.21}$$

Motion inevitably occurs if the inclination of the plane exceeds the angle of static repose (i.e. $\beta > \alpha$). The forces F_r and F_y are associated with the forces induced within the porous bed by the impingement of the jet, while the submerged weight is given by $W = C_b\rho(s-1)g$, where C_b is the concentration of particles in the bed. This allows the calculation of an erosion boundary by application of this stability criterion. This erosion boundary is denoted by $\eta = F(\xi)$ and hence in terms of dimensionless variables, the following relationship along the boundary is produced,

$$\left|\frac{F_r}{-F_y + \rho(s-1)C_b gH / p_0}\right|_{\eta=F(\xi)} \leq \tan(\alpha - \beta) \tag{5.22}$$

$$\text{where} \qquad \frac{\mathrm{d}F}{\mathrm{d}\xi} = -\tan\beta \tag{5.23}$$

These equations introduce a new dimensionless parameter, given by ($\Theta = p_0 / \rho(s-1)C_b gH$), which is a measure of the scale of the pressure gradients to the gravitational force and is similar, but not identical, to the Shields parameter introduced above. The parameter Θ expresses the ratio of the driving to the stabilizing forces, which act on the bed of particles and is henceforth referred to as the 'effective' erosion parameter, noting that if $p_0 = \rho u_*^2$ then $\Theta = \theta d / HC_b$.

This analysis illustrates how a downward moving event can mobilize particles. The turning moment about the point of contact with a neighbouring particle, arising from the radial force (F_r), can be sufficient to overcome the resisting moment arising from the vertical force on the particle, even though the incoming event may augment the weight of the particle.

The erosion boundary is calculated using Equations 5.18, 5.19, 5.22 and 5.23. Far from the stagnation point there is no excess pressure and so this boundary must return to the bed surface. The point at which this occurs ($\xi = \xi_*$) is found by enforcing the equality of Equation 5.22 and solving for $\tan\beta = 0$. The range between the stagnation point and this position is the extent of the influence of the jet. In order for such a position to exist, it is a requirement that $\Theta > \Theta_c$. Hence this imposes a critical value of the parameter Θ, below which it is assumed there is no bulk erosion. For $0 < \xi < \xi_*$, the equality of Equation 5.22 is enforced, whereas beyond this point the bed is assumed flat ($\beta = 0$) which permits the ratio of the driving and stabilizing forces to be less than $\tan\alpha$. The condition that the erosion boundary must lie below the initial boundary of the bed is also imposed.

The critical erosion parameter varies with the permeability of the bed, which is related to the grain size (Figure 5.6). It is found empirically that

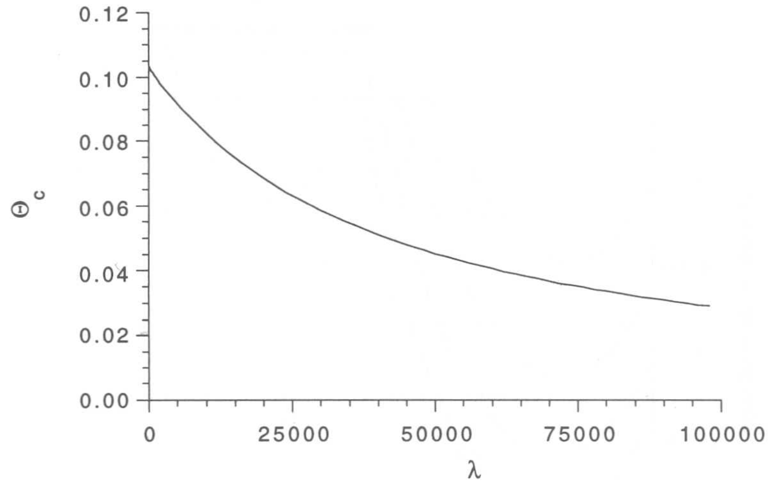

Figure 5.6. The variation of the critical erosion parameter (Θ_c) with the dimensionless permeability (λ)

$$\Theta_c \sim 0.1/\left((\lambda/40\,000)+1\right) \tag{5.24}$$

The following section discusses the connection between this critical value of the erosion parameter and the critical Shields parameter.

Profiles of the erosion boundary are calculated for various values of the erosion parameter and shown in Figure 5.7. For these profiles, λ is set equal to 1000. Two regions are elucidated by these erosion profiles. Close to the surface of the bed in a thin layer, the grains are mobilized by the action of shear. However, these effects are poorly transmitted through the bed and further from the surface the grains are mobilized by gradients of the induced pressure field.

If the eroded volume between the flat bed and the erosion boundary is calculated as a function of the effective erosion parameter (Figure 5.8), it is found that there is an approximately linear relation between the two, empirically given in dimensional form by

$$\text{Eroded Volume} = 0.001\pi\left(\Theta - \Theta_c\right)H^3 \tag{5.25}$$

This linear relationship may be rationalized by a simple scaling argument in each of the two regions of the flow within the porous bed. For the region in which the pressure gradients are the dominant force, $\partial Q/\partial\eta \sim \eta^{-3}$ and so by continuity $\partial Q/\partial\xi \sim \xi^{-3}$. Therefore, by enforcing the stability criterion (Equation 5.22), the erosion boundary $\xi, \eta \sim \Theta^{1/3}$ and hence the eroded volume is found to scale linearly with Θ. Conversely, within the boundary layer (of depth $1/\lambda$) adjacent to the bed

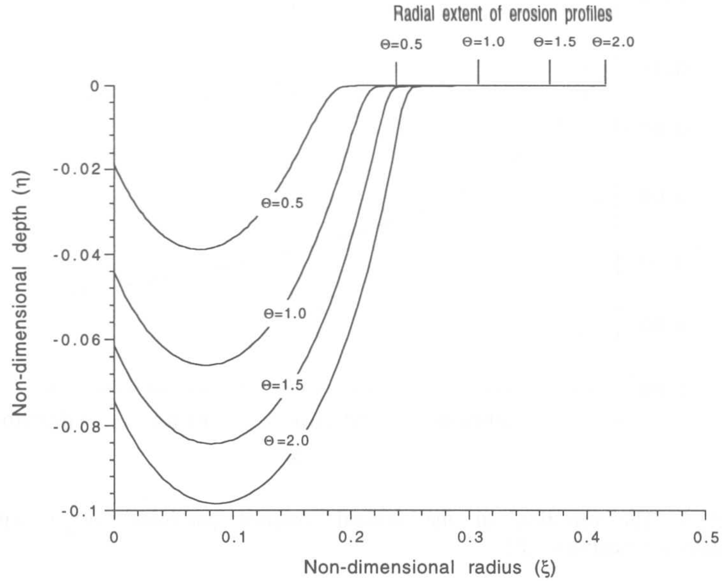

Figure 5.7. Theoretically calculated erosion boundary profiles for various values of the effective Shields parameter (Θ)

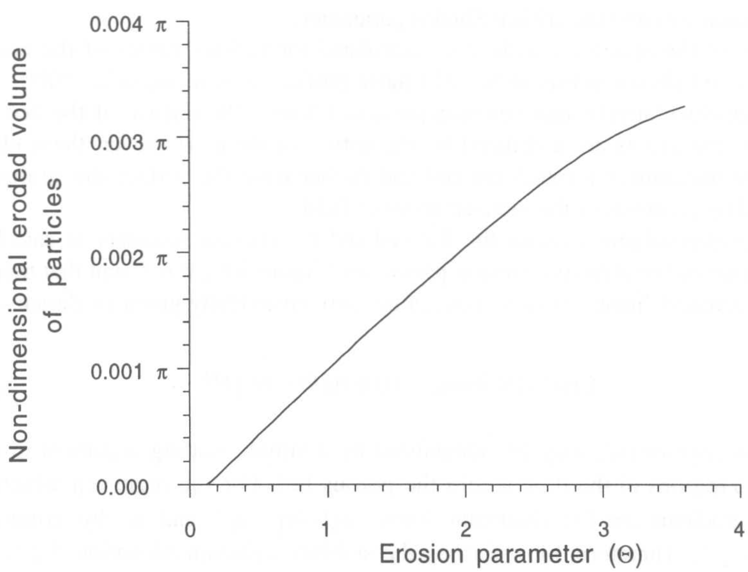

Figure 5.8. The variation of the dimensionless volume of particles with the erosion parameter (Θ)

surface, the shear stress decays as $O(1/\xi^2)$. Hence, the radius of the eroded, cylindrical volume is given by $\xi \sim \sqrt{\lambda\Theta}$ and so the volume within the boundary layer also scales linearly with Θ. These scaling arguments justify the claim that the functional dependence of the eroded volume is independent of the magnitude of the ratio of maximum pressure to shear stress (p_0/σ_0). Within both the pressure and shear stress dominated regions, the eroded volume of particles varies linearly with the erosion parameter (Θ). In the case shown here, $p_0/\sigma_0 >> 1$ and so the pressure dominated region is of more significance.

According to this simple reasoning, within neither the shear-dominated boundary layer, nor the pressure-dominated main body of the flow, does the eroded volume depend on the non-dimensional permeability. This fact is borne out by the numerical evaluation of the eroded volume for different permeabilities. A wide dispersion of grain sizes, however, may affect the friction angle ($\tan\alpha$), introducing a grain size effect in the mobility criterion and the shape of the eroded volume.

RATE OF BEDLOAD TRANSPORT

The above analysis is now used to develop an expression for the rate of bedload transport in equilibrium conditions and suggest a scaling relationship between the flux of bedload transport and the characteristics of the flow. More specifically, a relationship is proposed of the form:

$$q_{bl} = U_s C_s \Delta \tag{5.26}$$

where U_s, C_s are respectively the typical horizontal velocity and concentration of particles within the bedload layer of thickness Δ.

Most models for bedload transport (e.g. Engelund & Fredsøe, 1976) neglect an explicit description of the pick-up and deposition of particles and assume that the fluid shear stress on the boundary takes the critical value for incipient motion. The concentration of particles is then elucidated by use of an empirical dynamic friction condition which links the weight of the particles to the drag exerted on them. The present study instead proposes to assess the magnitude of the bedload transport in equilibrium conditions by assuming a balance between the volume of particles entrained and deposited.

The sweep event has been modelled by considering an axisymmetric jet of scale H and it is assumed that the jet is quasi-periodic with temporal period of recurrence T and areal frequency A, the area on the bed within which only one event may be found (Figure 5.9). The volume of particles picked up by a single event is simply related to the eroded volume from Equation 5.25 and is thus given by

$$V_e = 0.001\pi C_b H^3(\Theta - \Theta_c) \tag{5.27}$$

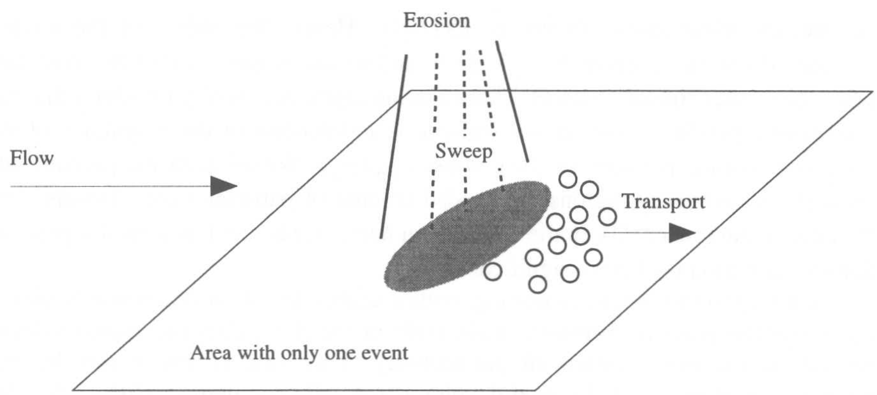

Figure 5.9. The turbulent, event-driven, picture of bedload transport. The sediment is mobilized by incoming sweep events which impinge upon the surface of the erodible bed, in-between which the particles are transported and deposited. The events are intermittent, yet repetitive flow structures within the turbulent boundary layer. Within an area of the bed (A), there is only one transport event

The volume of particles deposited during the period between events, throughout the area between events, is given by

$$V_d = \begin{cases} Aw_s TC_s, & \Delta \geq w_s T \\ A\Delta C_s, & \Delta < w_s T \end{cases} \tag{5.28}$$

where w_s is the settling velocity of the particles. These two scaling options correspond to whether or not all of the entrained particles are deposited before a subsequent event occurs. In equilibrium, the volume of entrained and deposited particles may be equated to find that

$$C_s = 0.001\pi C_b \frac{H^3}{Ah}\left(\Theta - \Theta_c\right) \tag{5.29}$$

where $h = \min\{w_s T, \Delta\}$. There has been considerable study to establish whether the scaling characteristics of sweep events depend upon viscous or inviscid processes (see the review of Cantwell, 1981). Blackwelder & Haritonidis (1983) found that the inter-event period is given by $T = 100\nu / u_*^2$, where ν is the kinematic viscosity of the fluid and u_* is the friction velocity. Conversely other investigators conclude that $T = 6\delta / u_\infty$, where δ is the extent of the boundary layer and u_∞ is the flow velocity at the edge of the boundary layer. The current study investigates a lower bound of the distance settled by a particle during the inter-event period using the scaling for this period proposed by Blackwelder & Haritonidis (1983) and the Stokes settling velocity of the particle $w_s = (s-1)gd^2 / 18\nu$. It is then found that

$$w_s T / d = 50 / 9\theta \tag{5.30}$$

The particle will therefore settle through many particle diameters during the inter-event period because the Shields parameter in the regime under consideration here is less than unity. However, observations suggest that the height of the bedload layer is only of the order of a few particle diameters. Hence it is concluded that $h = \Delta$, which is equivalent to assuming that all the mobilized particles settle back to the boundary before subsequent events occur. The rate of bedload transport is then given by:

$$q_{bl} = 0.001\pi \frac{U_s C_b H^3}{A}\left(\Theta - \Theta_c\right) \tag{5.31}$$

or in dimensionless form:

$$q_{bl} / \sqrt{(s-1)gd^3} = 0.001\pi \frac{U_s}{u_*}\frac{p_0}{\rho u_*{}^2}\frac{H^2}{A}\sqrt{\theta}\left(\theta - \theta_c\right) \tag{5.32}$$

where $\theta_c = \Theta_c H C_b \rho u_*{}^2 / p_0 d$. This expression is of the same functional form as that proposed by Nielsen (1992). The magnitude of the three ratios U_s / u_*, $p_0 / \rho u_*{}^2$ and H^2 / A, which represent the ratio of the horizontal velocity of the sediment to the friction velocity, the ratio of the stagnation pressure of the sweep event to the bed shear stress and the ratio of the square of the length-scale of the sweep event to the areal frequency, respectively, may now be estimated. Nielsen (1992) suggests that the typical velocity of the sediment within the bedload is approximately $5u_*$ and so it is assumed $U_s / u_* = 5$. Drake *et al.* (1988) calculated the area on the bed within their window of study which was affected by an individual sweep event, and assuming their study window corresponds to the area within which only one sweep event may be found, it can be estimated from their data that $H^2 / A = 0.3$. Finally from a field study of Start Bay, south-west England (Soulsby, 1983), it was found that sweep events contribute approximately 62% of the Reynolds shear stress in only 10% of the time and so the average shear stress contribution per event is approximately 6 times the overall time-averaged shear stress. The extent of a sweep event is fixed as when the shear stress exceeds the mean shear stress by a factor of 3 (Drake *et al.*, 1988) and hence an event is identified $0 < \xi < \xi_{sw}$, where $\sigma_s(\xi_{sw}) = 3\rho u_*^2$. The shear stress distribution (Equation 5.8) is then averaged over this interval and the average is required to be $6\rho u_*^2$ This sets $\sigma_0 / \rho u_*^2 = 12$ and thus $p_0 / \rho u_*^2 = 2 \times 10^3$. It is emphasized that both p_0 and σ_0 are the peak pressure and shear stress associated with the event. The pressure decays exponentially rapidly away from the point of impingement, while the shear stress decays algebraically. Using these estimates, Equation 5.30 is used to obtain the flux of bedload transport:

$$q_{bl} / \sqrt{(s-1)gd^3} = 9\sqrt{\theta}\left(\theta - \theta_c\right) \tag{5.33}$$

This formula is similar to the expressions for bedload transport which were semi-empirically derived by a number of other investigators (Engelund & Fredsøe, 1976; Nielsen, 1992). Furthermore, not only is the functional form of this relationship similar to previously suggested formulae, but also the crudely estimated constant, with a value of 9, compares favourably with previous studies.

These estimates may also be used to derive a value of the critical Shields parameter. Noting Equation 5.24 and that $\theta_c = \Theta_c H C_b \rho u_*^2 / p_0 d$, it is found that $\theta_c = 0.03$. This value is of the correct order of magnitude, but is slightly less than the usual critical value for the initiation of motion.

CONCLUSIONS

This paper has presented a framework for the analysis of turbulent, event-driven bedload transport by considering the impingement of individual sweep-like events upon a granular bed and calculating the volume of particles which are mobilized by an event. This volume depends both on the hydrodynamic characteristics of the incoming event and the size and density of the particle. Balancing the volume of particles which are mobilized with those deposited between events, allows formulation of an expression for the rate of bedload transport in equilibrium conditions. The expression which emerges from this new framework is in accord with those suggested by experimental studies and with those derived by existing theories. This result strongly suggests that models cannot neglect a description of event-driven transport. Indeed, the model presented here for event-driven bedload transport can account for a significant portion of material in transport for the range $\theta_c < \theta < 1$.

This framework has developed an expression for the pick-up function in bedload transport by appealing to the dynamics of the turbulent events and their interaction with the granular bed. The present study has not explicitly invoked the Bagnold assumption which requires the mean shear stress to be reduced to critical at the bed, nor has it utilized a dynamic coefficient of friction to relate the weight of the moving particles to the horizontal drag force exerted by the fluid. Rather these issues have been supplanted by an event-dominated view of the turbulent motion. The current model implicitly assumes that the flow is dominated by turbulent events close to the boundary and that the mean flow, onto which the event motions are superimposed, does not contribute to the mobilization of the sediment. Instead the mean flow does play a vital role in the advection of the mobilized sediment. This is analogous to sediment transport under combined waves and currents, in which the wave-motion mobilizes the particles and the current transports them. This study has focused on the most frequent turbulent events which render positive contributions to the Reynolds shear stress, namely ejection and sweep events. Critical conditions have been enforced at the bed by means of a stability criterion in which the contact friction is expressed through an angle of static repose and therefore elucidates the volume of particles mobilized as a function of the particle characteristics and the hydrodynamic characteristics of the incoming event. Consideration of the impact of sweep events on a granular bed can therefore potentially serve as a boundary condition for the turbulent event-driven suspension of particles, as described by Hogg *et al.* (1994).

It should be noted that the analytical framework presented here is based upon the notion of particle conservation; it has not considered the momentum transported by these downward moving, relatively fast events, nor the momentum associated with the mobilized particles. A more complete model would evaluate these contributions to the total shear stress within the flow and would enable the assessment of the particle velocity within the flow, rather than postulating that $U_s \sim u_*$. Furthermore, this model may be improved if it is driven statistically, taking into account the statistical distribution of the characteristic length and time scales of the impinging events. In addition, it may be possible to extend the analysis to non-equilibrium conditions or bedload transport driven by oscillatory flows. Nevertheless, the new model presented here has revealed how downward moving sweep events are capable of mobilizing particles in repose on the bed and has permitted derivation of an equilibrium transport formula which is in accord with experimental observations. Such turbulent event driven models can thus provide considerable insight to the process of bedload transport.

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APPENDIX

This appendix presents a derivation of the pressure distribution within a porous bed of particles which is driven by a distribution of pressure along the surface of the bed. The equations to be solved are Equations 5.9–5.12. An axisymmetric variation of the pressure is considered ($p \equiv p(r, y)$) and hence the equations reduce to:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{\partial^2 p}{\partial y^2}=0 \tag{5.34}$$

$$p \to 0, \quad y \to -\infty \tag{5.35}$$

$$p = p_s(r), \quad z = 0 \tag{5.36}$$

The Fourier-Bessel transform of the pressure field is taken with respect to r, assuming that the surface pressure decays sufficiently rapidly away from the stagnation point, so that the Fourier-Bessel transform of $p_s(r)$ exists. The transformed variables are denoted by $\tilde{p} = (k, y)$ and hence

$$\tilde{p}(k, y) = \tilde{p}_s(k)\exp(ky) \tag{5.37}$$

Inverting this transform, the standard result is found (Morse & Feshbach, 1953)

$$p(r, y) = \int_0^\infty k\exp(ky)J_0(kr)\int_0^\infty r'\, p_s(r')J_0(kr')\mathrm{d}r'\,\mathrm{d}k \tag{5.38}$$

where $J_0(r)$ is the zeroth-order Bessel function of the first kind. Substitution of the surface pressure distribution into this expression $p_s = p_0\exp(-\gamma r^2)$ yields

$$p(r, y) = \int_0^\infty k\exp(ky)\,J_0(kr)\exp(-k^2/4\gamma)/2\gamma\,\mathrm{d}k \tag{5.39}$$

This integral is then evaluated numerically to give the pressure distribution within the bed.

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NOTATION

a	diameter of jet nozzle
A	areal frequency of events
C	particle concentration within flow
C_b	particle concentration within granular bed
C_s	representative particle concentration in bedload layer
d	particle diameter

D	depth of flow
F_r	non-dimensional radial force within bed
F_y	non-dimensional vertical force within bed
g	gravitational acceleration
H	distance from jet nozzle to boundary
J_0	zeroth-order Bessel function of the first kind
p	pressure within granular bed
p_s	pressure on surface of the bed
p_0	pressure at stagnation point
q_{bl}	bedload flux of particles
Q	non-dimensional pressure within granular bed
r	radial distance from stagnation point
s	ratio of solid to fluid densities
T	temporal period of event recurrence
u	longitudinal velocity
u_*	friction velocity
u_∞	velocity at edge of boundary layer
U_0	jet velocity
V_d	volume of particles deposited between events
V_e	volume of particles entrained by event
w_s	particle settling velocity
W	submerged weight of particles
y	vertical distance from stagnation point
α	angle of repose
β	angle of sloping bed
δ_∞	boundary layer extent
Δ	thickness of bedload layer
η	non-dimensional vertical distance from stagnation point
θ	Shields' parameter
θ_c	critical Shields' parameter
Θ	erosion parameter
Θ_c	critical erosion parameter
λ	non-dimensional permeability of granular bed
Λ	permeability of granular bed
μ	dynamic viscosity
ν	kinematic viscosity
ξ	non-dimensional radial distance from stagnation point
ξ_{sw}	non-dimensional radial extent of sweep event
σ_b	shear stress on the lower boundary
σ_s	distribution of shear stress on the lower boundary
σ_0	maximum shear stress on the lower boundary