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GEOPHYSICS

Diapirism in a Rheologically Stratified Medium

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Sedimentary cover enclosing salt rocks is gravitationally unstable. Salt diapirs are formed owing to the fact that salt, being less dense than the overlying sedimentary rocks, tends to rise into the region occupied by these rocks. Diapirism in a viscous medium has been studied sufficiently (see, for instance, [1–4]). However, the rheology of sedimentary overburden is complicated and can be described by the model of a brittle or perfectly plastic medium. This work investigates the early stages of salt diapirism and analyzes the gravitational instability of a viscous salt layer overlapped by a perfectly plastic layer of sediments under horizontal compression or extension. Recent investigations in the region of salt tectonics have placed special emphasis on diapirism resulting from extension or compression of brittle overburden rocks (for instance, [5]). Effects of non-Newtonian power-law rheology on the medium instability were studied in [6–10]. However, these works either do not analyze the gravitational instability or do not show essential features characterizing the rheologically stratified medium instability. In the present work, we report the results of analytical investigations without the use of rigorous mathematical formulations.

1. Let us consider a structure consisting of a perfectly plastic layer (with a density of ρ_1 and an effective viscosity of μ_1 in the domain $[0; h_1]$) overlying a layer of viscous liquid with a density of ρ_2 and a viscosity of μ_2 in the domain $[-h_2; 0]$. The equations of moment, mass, and continuity conservation were used for describing the medium motions. The stress tensor τ_{ij} $(i, j = x, z)$ is related to the deformation velocity tensor ε_{ij} by the non-Newtonian power law [11]

$$\tau_{ij} = C\varepsilon^{\frac{1-n}{n}}\varepsilon_{ij},$$

where x and z are the horizontal and vertical coordinates, the constant C is determined from thermodynamic conditions, n is the power, and $\varepsilon = (\varepsilon_{xx}^2 + \varepsilon_{zz}^2 + 2\varepsilon_{xz}^2)^{\frac{1}{2}}$ is the second invariant of the deformation velocity. The effective viscosity μ of the power-law liquid is determined in this case as $\mu = 0.5C\varepsilon^{\frac{1-n}{n}}$. In two limiting cases, namely a viscous liquid and a perfectly plastic material, the power is equal to 1 and ∞, respectively.

It is assumed that the structure of layers is subject to horizontal extension (or compression), i.e., the basic flow, for which $\bar{\varepsilon}_{xx} = \gamma$ (or $-\gamma$ in the case of compression) and γ is a constant value, is specified in this case. Besides, $\bar{\varepsilon}_{zz} = -\bar{\varepsilon}_{xx}$ (due to incompressibility of the medium), and $\bar{\varepsilon}_{xz} = \bar{\varepsilon}_{zx} = 0$ (since we consider pure shear flow).

At the interface of two layers ($z = 0$), the conditions of continuity of velocity, shear stresses, and normal stresses are specified with due regard for the forces appearing due to the difference in densities and viscosities near the interface. For modeling various rheological situations at the upper boundary of the structure, both no-slip (adhesion) and stress-free conditions were considered. The no-slip conditions are specified at the lower boundary.

In order to study the growth of minor perturbations of the interface between the layers, let us represent physical variables (for example, the vertical velocity W) in the following form:

$$W(x, z, t) = w(z)\exp(ikx + pt), \quad (1)$$

where t is time, $k = \frac{2\pi}{L}$ is the wave number, L is the wavelength of perturbations, and p is the growth rate of perturbations. The stability problem is reduced to the

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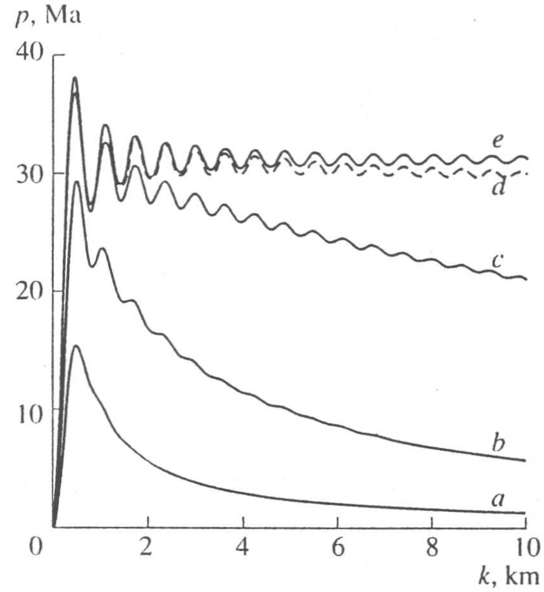

Fig. 1. The growth rate of diapirs p vs. the wave number k for different values of the ratio of effective viscosities ν: (*a*) 1; (*b*) 0.1; (*c*) 10^{-2}; (*d*) 10^{-3}; (*e*) 10^{-4} at $h_1/h_2 = 1$ and $\rho_1 - \rho_2 = 300$ kg m^{-3} in the case of rigid upper boundary of the structure.

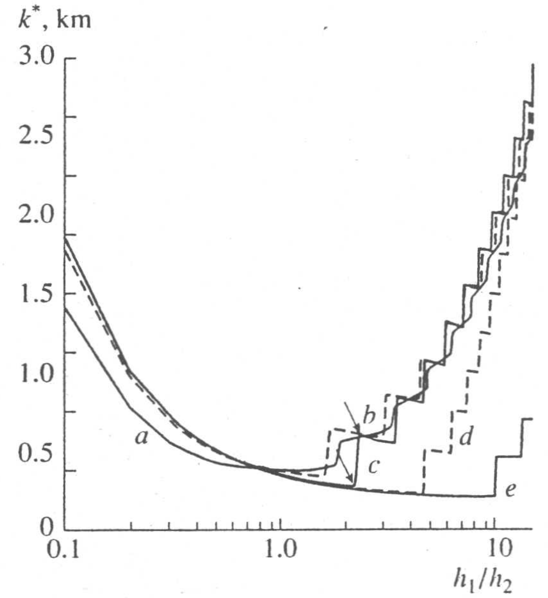

Fig. 2. The predominant wave number k^* vs. the ratio of layer thicknesses h_1/h_2 for different values of the ratio of effective viscosities ν: (*a*) 1; (*b*) 0.1; (*c*) 10^{-2}; (*d*) 10^{-3}; (*e*) 10^{-4} at $\rho_1 - \rho_2 = 300$ kg m^{-3} in the case of rigid upper boundary of the structure. The arrows indicate a stepwise change in k^*.

analysis of the growth rate of perturbations p for various values of the wavelength of perturbations L or the wave number k, the ratios of effective viscosities of the layers $\nu = \frac{\mu_2}{\mu_1}$, the ratios of layer thicknesses $\frac{h_1}{h_2}$, and the difference of densities $\rho_1 - \rho_2$, It is clear from (1) that if the p value is such that its real part is positive, then the layered structure will be unstable. If such p values are absent, then stability will take place. In order to illustrate the results of the analysis of a stratified system instability, the following values of parameters characterizing the salt complex were chosen: $h_1 + h_2 = 10$ km; $\mu_1 + \mu_2 = 2 \times 10^{20}$ Pa s; $\rho_1 = 2.5 \times 10^3$ kg m^{-3}; and $\rho_2 = 2.2 \times 10^3$ kg m^{-3} are the densities of sedimentary overburden and of salt, respectively.

2. Let us consider the case when extension or compression of the medium is insignificant ($\gamma < 10^{-17}$ s^{-1}). In this case, the gravitational instability plays a crucial role in the development of medium deformations. The no-slip conditions are specified at the upper boundary of the layered structure. The p versus k dependence is presented in Fig. 1 for various values of the ratio of the salt viscosity μ_2 to the effective viscosity of sedimentary overburden μ_1. The sinuosity of curves characterizing the growth rate of perturbations in the structure is determined through the solution of a hyperbolic equation describing velocity perturbations. It is seen from Fig. 1 that when the effective viscosity of the upper layer does not exceed that of the lower layer by very much, the instability of the layer structure is governed by viscous motions; i.e., there exists a mode of maximum instability and the growth rate of perturbations diminishes at large and small wavelengths of these perturbations. However, when the effective viscosity of sedimentary overburden exceeds that of salt by several orders of magnitude, the situation changes and the instability is governed by plastic deformations; i.e., the curves of perturbation growth acquire a strong sinuosity and, for small wavelengths, the growth rate of perturbations asymptotically approach the following constant:

$$p = 0.25(\rho_1 - \rho_2)g(h_1 + h_2)/(\mu_1 + \mu_2).$$

Thus, short-wave perturbations in this structure will grow exponentially and increase by a factor of 2.7 during 870 ka. In addition, as can be seen from Fig. 1, the growth rate of perturbations p has a maximum value that does not differ substantially from values of this indicator at other peaks of the growth rate curve. Hence, the initial perturbation of the salt–overburden interface can produce an assemblage of diapirs with a variable wavelength rather than diapirs with a clearly predominant wavelength, as in the case of the instability of viscous layer systems. This fact can serve as an explanation for the spatially inhomogeneous distribution of salt diapirs.

The predominant wave number corresponding to the maximum value of the growth rate of perturbations first diminishes and then increases again as a consequence of a series of jumps (Fig. 2). These jumps are associated with the sinuosity of the perturbation growth curve and occur at the moment at which the second, third, etc.,

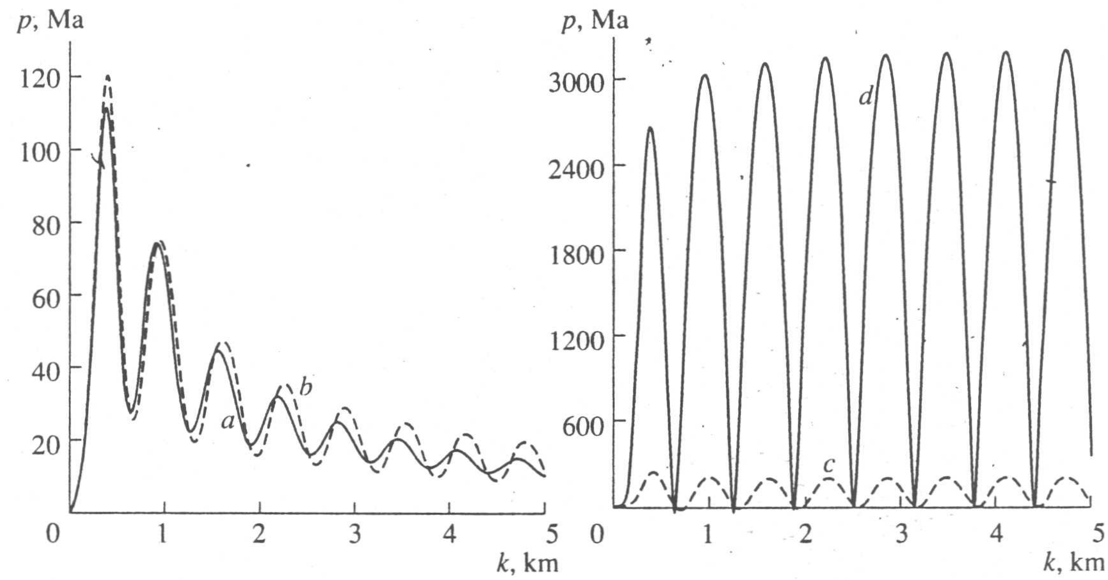

Fig. 3. The growth rate of diapirs p vs. the wave number k in the case of a stress-free upper boundary of the layered structure for $\nu = 0.1$, $h_1/h_2 = 1$, $\rho_1 - \rho_2 = 300$ kg m^{-3}, and different values of the deformation velocity of the basic flow γ: (*a*) 10^{-17} s^{-1}; (*b*) 10^{-16} s^{-1}; (*c*) 10^{-15} s^{-1}; (*d*) 10^{-14} s^{-1}.

peaks of the curve become maximum (i.e., higher than the neighboring peaks). Since the length of troughs is governed by the predominant wavelength of perturbation, the predominant wave number increase can explain the reason for small interdomal distances in some salt provinces (for example, diapiric uplifts in Iran [12]).

3. Further, the influence of extension or compression on the gravitational instability of the layered structure has been studied. Figure 3 illustrates the curves of perturbation growth in the medium under horizontal compression in the case of a stress-free upper boundary of the structure of the layers. The values of the growth rate of perturbations p are positive for small values of the compressive deformation velocity γ. The maximum value of the indicator p increases with the growth of γ. At large values of the deformation velocity, the system passes to the resonance state characterized by a linear dependence of the growth rate of perturbations on the ratio of effective viscosities ν [8]. We found the asymptotic expression for the growth rate of perturbations at minor values of ν and wavelengths:

$$p = (\gamma/\nu)\sin(k[h_1 + h_2]).$$

The indicator p can assume both positive and negative values depending on the γ sign (i.e., extension or compression applied to the layered system) and on the sign of the sine function. It attains maximum values $p_{\max} = \mathrm{abs}(\gamma)/\nu$ at the wave numbers $k[h_1 + h_2] = \pi/2 + \pi m$, $m = 1, 2, \ldots, \infty$. Hence, the instability appearing under horizontal extension or compression of the layered structure replaces the gravitational instability for the deformation velocity values of the basic flow γ exceeding 10^{-16} s^{-1}.

Thus, considering that the effective viscosity of the sedimentary overburden exceeds that of salt by far, one can conclude that the gravitational instability of a rheologically stratified structure is governed by the behavior of a perfectly plastic material. Although the rheology of a sedimentary complex is much more complicated than the rheology of a perfectly plastic material, the investigation of a simple rheologically stratified structure represents a substantial step towards the understanding of deformation processes in sedimentary rocks.

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