PERGAMON

Journal of Geodynamics 31 (2001) 447–458

JOURNAL OF GEODYNAMICS

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# Analytical modelling of viscous diapirism through a strongly non-Newtonian overburden subject to horizontal forces

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Received 27 November 2000; received in revised form 1 March 2001; accepted 10 April 2001

## Abstract

We study the early stages of diapirism and analyse the gravitational and buckling instabilities of a buoyant viscous layer overlain by a layer of strongly non-Newtonian power-law rheology (when a power-law exponent tends to infinity). This situation models rocksalt under a layer of a perfectly plastic overburden. The growth rate of small perturbations on the interface between the two layers and the wavelength of the most unstable perturbations are found and compared with those of structures consisting of two Newtonian or two strongly non-Newtonian viscous layers. Effects due to the effective viscosity and thickness ratios between the two layers are assessed. Considering the effective viscosity of the overburden to be much greater than the viscosity of the buoyant salt layer, we obtain the following results. In the case of simple gravitational instability and no-slip boundary conditions, the instability pattern is similar to that in a strongly non-Newtonian power-law material. An increase in the thickness of the overburden decreases the dominant wavelength of the most unstable mode, while the dominant wavelength is lengthened in the case of Newtonian viscous layers. When the system of layers is subjected to either horizontal extension or shortening, and the upper boundary of the system is stress-free, the buckling instability overwhelms the gravitational instability, and the dynamic growth rate of the instability depends linearly on the effective viscosity ratio. We conclude that the introduction of strongly non-Newtonian power-law rheology into diapir overburdens greatly affects instability parameters such as growth rate and dominant wavelength of perturbations and hence, alters interdiapir spacings. © 2001 Elsevier Science Ltd. All rights reserved.

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PII: S0264-3707(01)00011-4

## 1. Introduction

Two fundamental types of instability are of considerable importance in the evolution of geological structures: gravitational and buckling instabilities. The gravitational instability is associated with inversions in density due to chemical or thermal heterogeneities, while the buckling instability arises from variations of viscosity under the action of an applied stress. The theory of gravitational instability predicts a negative growth rate for any layered structure in the absence of density inversion and a positive one for a structure with a density inversion (Chandrasekhar, 1961). Any stress-induced secondary flow produces a positive dynamic growth rate (Smith, 1975, 1977). The two effects compete to determine whether any disturbance will grow or decay.

Density inversions are common in nature, for example, when evaporite deposits (e.g. rocksalt) are buried under compacting clastic sediments in depressions of the Earth. Overburdens thicker than about 1 km can become more dense than the evaporite, which can then rise through the overlying layer forming diapirs (Talbot et al., 1991). Two types of diapiric structures are distinguished: upbuilt and downbuilt diapirs (Jackson and Talbot, 1994). The first one represents an active piercement of salt into its overburden (i.e. upbuilding), during which all the overburden was in place before halokinesis began. As the diapir rises, its base remains at a constant depth below the sedimentary surface, while its crest rises toward the surface. The second type of diapiric structures represents passive diapir growth (i.e. downbuilding), in which accumulation of a thickening overburden depresses the underlying salt while the top of salt in the diapir stays near the depositional (top) surface.

Studies of natural diapirs have benefited from theoretical analyses, physical and numerical modelling based on the Rayleigh–Taylor instability of viscous layers. Thus, for example, a theory of gravitational instability of layered geological media was developed by Biot (1965), Biot and Odé (1965) and Ramberg (1968) for the case of small perturbation under various complications (e.g. variable viscosity, variable thicknesses of layers, compaction). The effect of the viscosity contrast between layers on the shape and growth rate of diapirs was studied numerically by Woidt (1978). Schmeling (1987) demonstrated how the dominant wavelength and the geometry of the gravity overturns are influenced by the shape of the initial perturbation. Lister and Kerr (1989) analysed the gravitational instability of viscous fluids, highlighting the dependence of the spacing and initial growth rate of diapirs on the geometry of the buoyant structure. Poliakov et al. (1993) and Naimark et al. (1998) studied, numerically, the effects of differential loading of sediments on diapirism. Physical models of the gravitational and buckling instabilities of viscous layered structures provided us with an accurate information on the kinematics (rather than the dynamics) of the structures in three dimensions (see, e.g. Ramberg, 1981).

In all these studies the overburden was considered to be a viscous fluid, whereas the rheological behavior of natural overburdens is more complex and is better described by a non-Newtonian fluid (Weijermars et al., 1993). In the case of non-Newtonian power-law fluids the stress tensor $\tau_{ij}$ $(i,j=x,z)$ and strain-rate tensor $\dot{\varepsilon}_{ij}$ are related by

$$\tau_{ij} = C\dot{\varepsilon}_{ij}\dot{\varepsilon}^{\frac{1-n}{n}}, \tag{1}$$

where $C$ is a proportionality factor defined from the thermodynamical conditions, $n$ is a power-law exponent, and $\dot{\varepsilon} = \left(\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{zz}^2 + 2\dot{\varepsilon}_{xz}^2\right)^{\frac{1}{2}}$ is the second invariant of the strain-rate.

Because the effective viscosity of most overburdens is very high, the deformation of the overburden is no longer controlled by dislocation creep, instead it is determined by a movement of blocks of the overburden along pre-existing faults (slip-lines) of various orientations. The dynamic friction along such faults does not depend upon the strain rate, and such a physical mechanism results in the rheological model of a perfectly plastic material which does not exhibit work-hardening but flows plastically under constant stress. Hence the stress–strain relationship for the overburden is obeying the von Mises equations (Prager and Hodge, 1951)

$$\tau_{ij} = \kappa\dot{\varepsilon}_{ij}/\dot{\varepsilon}, \tag{2}$$

where $\kappa$ is the yield limit. The second invariant of the stress, $\tau = (\tau_{kl}\tau_{kl})^{\frac{1}{2}}$, equals the yield limit, $\kappa$, for any strain rate. When $\tau < \kappa$, there is no plastic deformation and hence no motion along the faults. A comparison of Eqs. (1) and (2) shows that the perfectly plastic material can be modelled as a strongly non-Newtonian power-law fluid at $n \to \infty$.

An objective of our research is to analyse the gravitational instability of a buoyant viscous layer overlain by a layer of strongly non-Newtonian power-law rheology in order to explain the non-uniform distribution of salt diapirs in many salt-bearing sedimentary basins. Several analytical and numerical investigations have been performed to discover the differences in growth rates of the gravitational instability in a viscous layered structure and in a structure containing layers of different rheology. The gravitational instability of stratified viscoelastic structures for both incompressible and compressible materials was analysed by Naimark and Ismail-Zadeh (1989, 1994) and Ismail-Zadeh (1994). Recently, Conrad and Molnar (1997) studied the gravitational instability of non-Newtonian layered structures. In addition, a few numerical models have been developed to explore the effects of rheological stratification on diapirism (Podladchikov et al., 1993; van Keken et al., 1993; Daudré and Cloetingh, 1994). However, these studies did not pursue the idea of rheological stratification sufficiently far to determine the full implications.

Another objective of our research is to investigate the effects of horizontal extension or shortening on the stability of rheologically layered structures. This objective is associated with recent advances in salt dynamics, highlighting the role of horizontal stretching or squeezing of a brittle overburden in the formation of salt structures (Vendeville and Jackson, 1992; Jackson and Talbot, 1994; Jackson and Vendeville, 1994). A few physical models were developed to study the effects of lateral movements on salt diapirism where an overburden was considered to be either non-Newtonian or frictional materials (Koyi, 1988, 1998). A series of papers (Fletcher, 1974; Smith, 1977, 1979; Fletcher and Hallet, 1983; Ricard and Froidevaux, 1986; Zuber et al., 1986; Martinod and Davy, 1992; Birger, 1996; Conrad and Molnar, 1997) discuss non-Newtonian effects on finite-amplitude tension and compression, but these papers either have not addressed the problem of the gravitational instability due to density inversion or have not evaluated all features of the buckling instability in rheologically stratified material.

In this paper we analyse growth rates of small perturbations of the rheologically layered structure, their dependence on effective viscosity and thickness ratios, and applied boundary conditions. Also we study effects of horizontal extension and shortening on the growth rates of perturbations on the interface between these layers. Exact analytical solutions are obtained for the problem considered; the asymptotical behavior of growth rate of perturbations are found for large effective viscosity ratio and small and large wavelength of the perturbations. We present, but do not derive here, results obtained analytically and discuss them in the subsequent sections.

## 2. Formulation of the analysis

We study the gravitational instability of a buoyant viscous layer of density $\rho_2$ and viscosity $\mu_2$ in $-h_2 \leqslant z \leqslant 0$ overlain by a strongly non-Newtonian viscous layer (power-law exponent tends to infinity) of density $\rho_1 > \rho_2$ and effective viscosity $\mu_1$ in $0 \leqslant z \leqslant h_1$ (see Fig. 1). We consider the layered model to be subject to horizontal extension or shortening, so that there is a basic background pure shear flow. The horizontal forces, acting along the $x$-axis, induce a horizontal strain-rate, $\dot{\bar{\varepsilon}}_{xx} = \gamma$, where $\gamma$ is a constant (defined to be positive in the case of extension and negative in the case of shortening). By continuity, the vertical strain-rate $\dot{\bar{\varepsilon}}_{zz} = -\gamma$. The remaining component of the strain-rate tensor, $\bar{\varepsilon}_{xz}$, is assumed to be zero.

The governing equations of motion are represented by the equations of rheology, continuity, momentum, and incompressibility (Chandrasekhar, 1961). Motivated by the extremely large viscosities of geological materials, we assume that inertial terms in the momentum equation are negligible. In order to obtain equations for the initial growth of small perturbations to the background state, we neglect all products and powers of the perturbations in the governing equations and retain only linear terms.

In the case of non-Newtonian power-law rheology the perturbations of components of stress tensor, $\delta\tau_{ij}$ $(i,j = x,z)$, and of strain-rate tensor, $\delta\dot{\varepsilon}_{ij}$, are presented in the following form (see, e.g. Smith, 1977)

$$\delta\tau_{xx} = \frac{2}{n}\bar{\mu}\delta\dot{\varepsilon}_{xx}, \quad \delta\tau_{zz} = \frac{2}{n}\bar{\mu}\delta\dot{\varepsilon}_{zz}, \quad \delta\tau_{xz} = 2\bar{\mu}\delta\dot{\varepsilon}_{xz}.$$

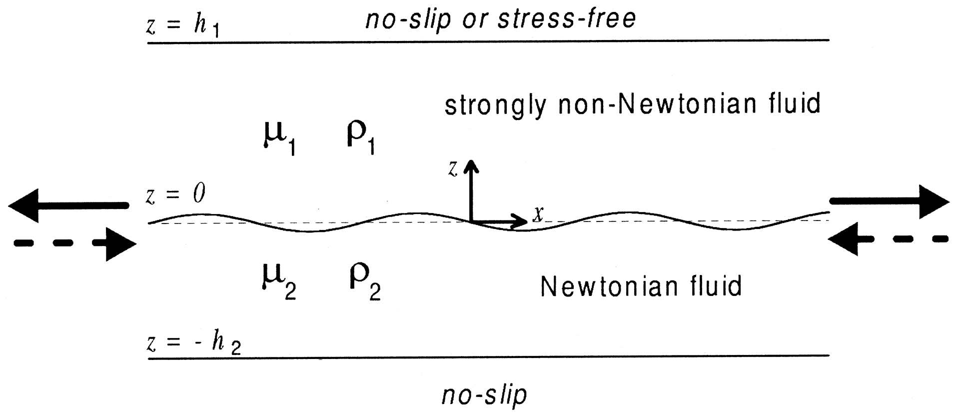


Fig. 1. A sketch for our analytical model of diapirism. A small sinusoidal perturbation is prescribed to the salt/overburden interface. $\mu_1$ and $\rho_1$ are the effective viscosity and density of the overburden; $\mu_2$ and $\rho_2$ are the viscosity and density of the salt layer. The layers are subject to horizontal extension or shortening (solid and dashed arrows).

Here the effective viscosity $\bar{\mu}$ is defined as $C\dot{\bar{\varepsilon}}^{\frac{1-n}{n}}$, where $\dot{\bar{\varepsilon}} = \left(\dot{\bar{\varepsilon}}_{xx}^2 + \dot{\bar{\varepsilon}}_{zz}^2\right)^{\frac{1}{2}}$ is the second invariant of the strain-rate for the basic background shear flow. For strongly non-Newtonian power-law (upper layer) and Newtonian (lower layer) fluids the effective viscosities are represented as $\mu_1 = C_1\dot{\bar{\varepsilon}}^{-1}$ and $\mu_2 = C_2$, respectively, where $C_1$ and $C_2$ are proportionality factors.

The conditions at the interface between the layers, $z=0$, follow from continuity of velocity, shear and normal stress allowing for the forces due to the density and viscosity discontinuities at the interface. In order to model different situations, the conditions we consider at the upper boundary are either no-slip or stress-free and at the lower boundary are no-slip.

We analyse the perturbations into normal modes, so that the vertical velocity $W(x,z,t)$ is represented as $w(z)\exp(ikx+pt)$, where $t$ is time, $k$ is the horizontal wavenumber ($=2\pi$/wavelength), and $p$ is the growth rate. The stability problem then reduces to determination of $p$ as a function of $k$. If $p$ has a negative real part for all $k$, then the system of layers is stable. If $p$ has a positive real part for some range of $k$ then the system is unstable, and a displacement of an interface grows exponentially with time.

Substituting the solutions to the perturbation equations for each layer into the boundary conditions, we derive a system of linear equations for the constants entering into the solutions. The growth rates (or eigenvalues of the system) are found from the resulting set of linear equations. Although there are as many eigenvalues as there are interfaces of the structure, one eigenvalue corresponds to a fastest growth of the perturbations. This is referred to as the maximum growth rate, and the corresponding wavenumber (or wavelength) is referred to as the dominant wavenumber (or wavelength). The methodology is described in detail by Chandrasekhar (1961).

## 3. Results of the analysis

Here we discuss only the results of geological relevance and omit the mathematical details which are presented in a separate paper (Ismail-Zadeh et al., 2001). To illustrate the results we take the following values of the model parameters: $h^* = 10$ km is the thickness of the layered structure, $\mu^* = 2\times10^{20}$ Pa s is the typical viscosity of sediments, $\rho_1 = 2.5\times10^3$ kg m$^{-3}$, and $\rho_2 = 2.2\times10^3$ kg m$^{-3}$, where $\rho_2$ and $\rho_1$ are the typical densities of rocksalt and its overburden, respectively.

First, we consider the case where the background strain rate is rather small ($\gamma < 10^{-17}$ s$^{-1}$) so that any buckling is slow to develop. In this case the gravitational instability acts independently of the background pure shear $\gamma$. The ($p$, $k$)-relationships are illustrated in Fig. 2A for various values of thickness ratio, $h_1/h_2$. The growth rate, $p$, and wavenumber, $k$, have dimensions of 1/time and 1/length, respectively. The relation $t=1/p$ is used to convert the growth rate to the characteristic time, that is, the time taken for the diapir growth by a factor $e$. The waviness of the growth rate curves is due to the fact that the perturbation equation for strongly non-Newtonian power-law materials at $n \to \infty$ is a hyperbolic wave equation (Bers et al., 1964) and the vertical velocity structure $w(z)$ is oscillatory. For comparison purposes, the perturbation equation for Newtonian fluids is an elliptic equation.

The dominant wavenumber initially decreases with increasing thickness ratio but then increases again by a series of abrupt jumps (Fig. 2B). This behavior is associated with the waviness of the growth rate curve (and hence is due to strongly non-Newtonian rheology of the upper layer) and

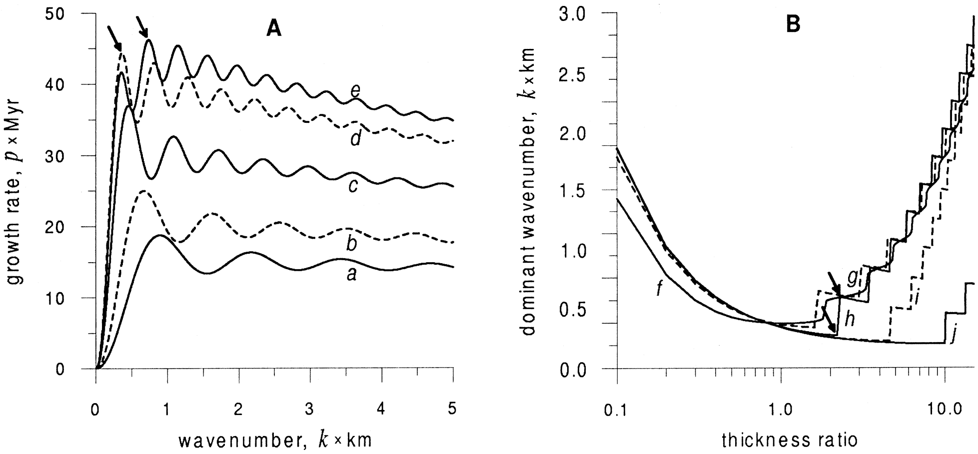


Fig. 2. Effect of the thickness ratio, $h_1/h_2$, on the growth rates of diapirs, $p$, and dominant wavenumbers, $k$. The upper boundary of the model is no-slip. Density contrast is taken to be 300 kg m$^{-3}$. (A) The growth rate versus wavenumber for the various values of the thickness ratio: *a*. 1/3; *b*. 1/2; *c*. 1; *d*. 2; and *e*. 3 at $\nu = 0.01$. (B) The dominant wavenumber versus the thickness ratio for the various values of the effective viscosity ratio, $\nu$: *f*. 1; *g*. 0.1; *h*. $10^{-2}$; *i*. $10^{-3}$; and *j*. $10^{-4}$. Arrows show the abrupt change of the dominant wavenumber that corresponds to the maximum growth rate.

it occurs when the second, third and so on peaks of the growth rate curve become higher than the surrounding peaks. The result obtained is valid for both a no-slip and a stress-free upper boundary. Arrows in Fig. 2 indicate how the dominant wavenumber becomes larger by an abrupt change.

Where $\nu = \mu_2/\mu_1$ is sufficiently small (the viscosity of salt is several orders of magnitude less than the effective viscosity of its overburden) and the model upper boundary is no-slip, we find that the growth rate approaches $p = 0.25(\rho_1 - \rho_2)gh^*/\mu^*$ for large wavenumbers. This means that short-wavelength perturbations will grow exponentially and increase by a factor of 2.7 in amplitude in about 0.83 Ma. Whilst there is a mode of maximum instability for which the amplitude of the disturbance grows most rapidly, other modes associated with a waviness of growth rate curves have almost the same growth rate for large wavenumbers (small wavelengths), when $\mu_1 \gg \mu_2$. Hence, initial perturbations in the salt/overburden interface may generate diapirs with a mixture of different wavelengths rather than diapirs with one dominant wavelength associated with a well-defined maximum growth rate. This offers a possible origin for a non-uniform distribution of mature diapirs.

In the case of stress-free conditions at the upper boundary of the model, the positive growth rate has a maximum at the dominant wavelength, and the rate is higher than the rate found in the case of no-slip conditions. This has an obvious interpretation: a stress-free surface is less resistant to deformation than a rigid one. The smaller the effective viscosity ratio $\nu$, the higher the positive growth rate and the larger the amplitude of the curve waviness.

We have also analysed models composed of two viscous layers and of two strongly non-Newtonian power-law layers in order to compare the growth rates of the gravitational instability with that for the rheologically layered model. A comparative analysis of such models shows that the behavior of a viscous layer overlain by a layer of strongly non-Newtonian power-law rheology depends on the effective viscosity ratio $\nu$ as depicted in Fig. 3. When the effective viscosity of the

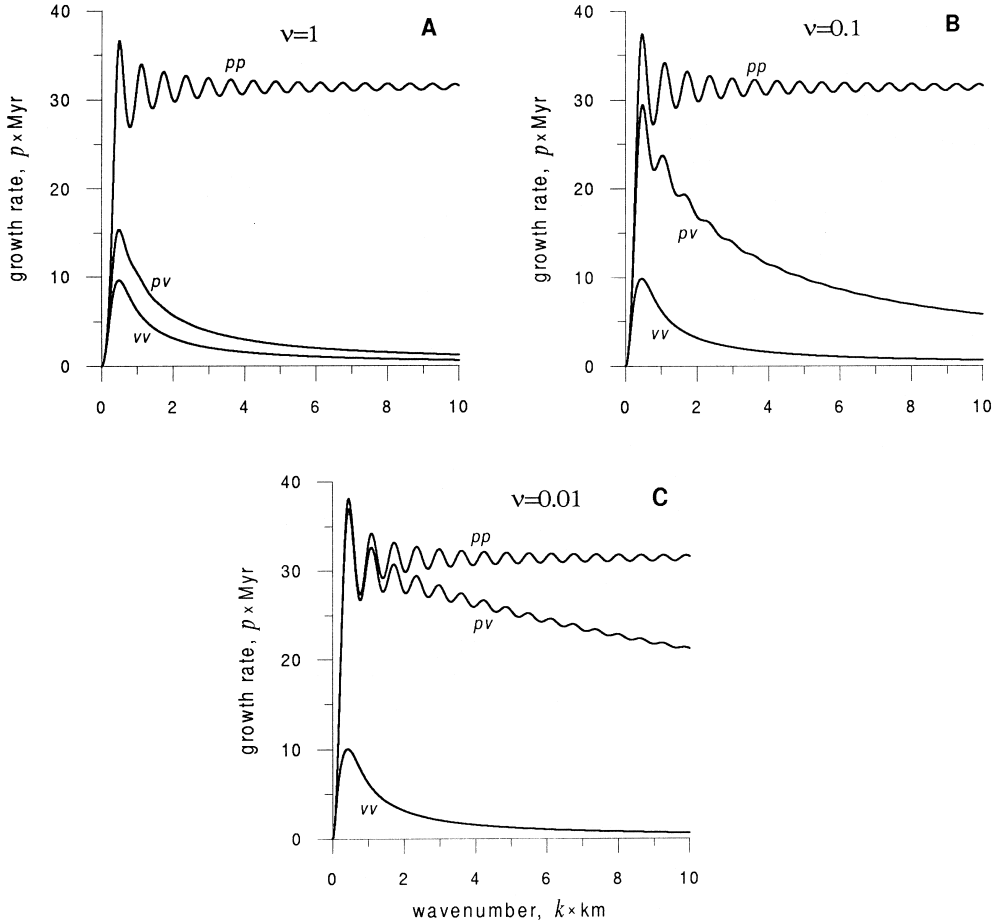


Fig. 3. The growth rate of diapirs versus wavenumber for various values of the effective viscosity ratio, $\nu$: A. 1; B. 0.1; and C. 0.01 at $h_1/h_2 = 1$ and $\rho_1 - \rho_2 = 300$ kg m$^{-3}$ in three cases of (*pv*) a viscous salt layer overlain by a strongly non-Newtonian sediments, (*pp*) a composition of two strongly non-Newtonian layers, and (*vv*) a composition of two viscous layers. The upper boundary of the model is no-slip.

upper layer is less than the viscosity of the lower layer, the layered structure behaves as though it consists of viscous layers.

Thus, there is a mode of maximum instability, and the growth rate vanishes for both small and large wavelengths. On the other hand, when the effective viscosity of the upper layer is much greater than the viscosity of the lower layer, the instability of the structure is similar to that of strongly non-Newtonian layers, that is, the growth rate curve oscillates with wavelength and approaches a specific constant at small wavelengths.

Second, we study the effects of interaction between gravitational and buckling instabilities for the case of rheological stratification by considering larger values of $\gamma$. The growth rate of any gravitational instability, driven by the density difference, is augmented by a buckling instability, driven by compression or tension of layers of different viscosity, and kinematic deformation of any perturbation by the strain rate $\gamma$.

There are four cases reflecting the effects of gravitational and buckling instability: (i) the layered system is subjected to horizontal compression in the presence of a density inversion; (ii) the system is subjected to horizontal tension in the presence of a density inversion; (iii) the system is subjected to horizontal compression in the absence of a density inversion; and (iv) the system is subjected to horizontal tension in the absence of a density inversion. Fig. 4 illustrates the growth rates of small perturbations of the layered structure in the first case (folding + density inversion). We see that the growth rates are positive for small values of the background strain rate, and the maximum growth rate increases in amplitude with an increase in the background strain rate.

In the second case the growth rate are positive or negative depending on values of viscosity ratio and of the basic strain rate. The maximum growth rate decreases in amplitude with a growth of basic background strain rate, $\gamma$, while the other sinusoidal peaks of the growth rate curve increase rapidly with a growth of $\gamma$. In the third and fourth cases, the total growth rates are negative for the small $\gamma$ and $\nu$, but then become either positive or negative depending on values of viscosity ratio and basic strain rate. For the all cases considered, at large basic strain rates the structure tends to "resonance" behavior (Smith, 1979) marked by a linear dependence of the growth rate on effective viscosity ratio $\nu$.

We find that

$$p_{\pm} = \pm\frac{\gamma}{\nu}\sin(kh^{*})$$

if $\nu \ll 1$ and sufficiently large wavenumbers $k$. This agrees with the results of Smith (1979) who showed a similar linear dependence in the case of folding and negligible gravity instability. The growth rate defined by the eigenvalue $p$ can be positive or negative depending on the signs of $\gamma$ and the trigonometric factor, and it attains a maximum value

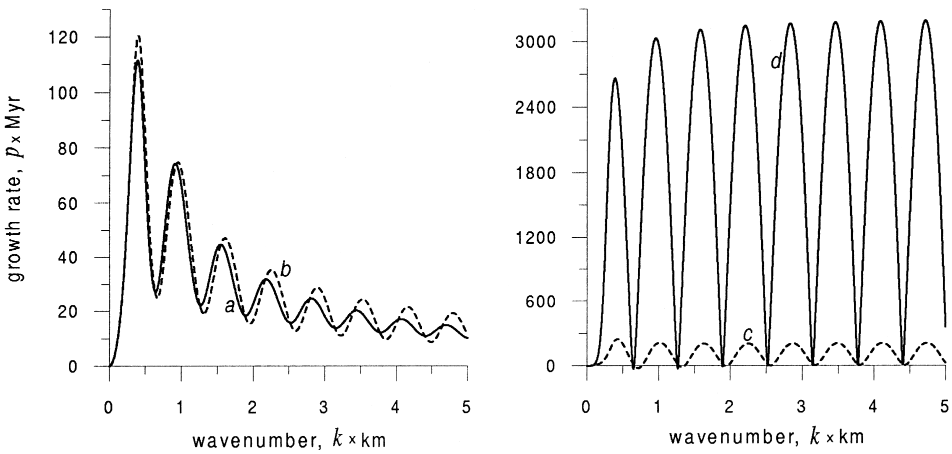


Fig. 4. The growth rate of diapirs versus wavenumber at $\nu = 0.1$, $h_1/h_2 = 1$ and $\rho_1$-$\rho_2 = 300$ kg m$^{-3}$ for the various values of the background strain rate, $\gamma$: $a$: $-10^{-17}$ s$^{-1}$; $b$: $-10^{-16}$ s$^{-1}$; $c$: $-10^{-15}$ s$^{-1}$; and $d$: $-10^{-14}$ s$^{-1}$. The upper boundary of the model is stress-free.

$$p_{\max} = \frac{\mathrm{abs}(\gamma)}{\nu}$$

at $kh^* = \pi/2 + \pi m$, $m = 1, 2, ..., \infty$. Therefore, the buckling instability replaces the gravitational instability during rapid horizontal shortening or extension of the rheologically layered structure.

In summary, the layered structure behaves like a viscous system at large values of effective viscosity ratio, $\nu$, and small values of basic background strain rate, $\gamma$, and approaches the behavior of a strongly non-Newtonian material at sufficiently small values of $\nu$ and large values of $\gamma$.

## 4. Discussion and conclusions

This section discusses the implications of our results for geological modelling and present conclusions about the results. This study is of considerable relevance to problems concerning the evolution of salt structures in sedimentary basins. Our analysis can be also applied to the problems of instability of brittle and ductile layers in the crust (Ranalli, 2000).

Although the shapes and patterns of salt structures can be very complex, the basic physical phenomena of diapirs can readily be explained by the gravitational instability of lighter salt underlying denser overburdens. If the interface between the two layers is sufficiently disturbed, the underlying low density salt flows upward due to the density inversion. The growth rate of a salt diapir actively upbuilding through its overburden depends on the density and viscosity contrasts, the thicknesses of the two media, the boundary conditions, and any applied extension or shortening (Schmeling, 1987; Talbot and Jackson 1987; Talbot et al., 1991).

We analysed the instability of a buoyant viscous layer underlying a dense strongly non-Newtonian power-law layer with a thickness that increases in time. During such sedimentary downbuilding, the overburden thickens with respect to the initial thickness of salt layer. We assume here that the sedimentation is rapid compared to the timescale of diapirism. Once a dominant wavelength is established, adding further overburden (e.g. by downbuilding of overburden) is not likely to change the wavelength. Hence the spacing between downbuilt diapirs (the distance between the crests of two neighbouring diapirs) will be defined by the dominant wavelength ($2\pi/k$). The analytical results show that the dominant wavelength is short when the overburden is thin, and with a large effective viscosity ratio $\nu$ it increases in length as the overburden thickens. When the effective viscosity of a strongly non-Newtonian overburden is rather high ($\nu$ is sufficiently small), the dominant wavelength tends to diminish when the thickness of the overburden exceeds the salt thickness. This may explain the surprisingly small distance between salt diapirs relative to the thickness of salt and overburden in the Great Kavir in Iran (Jackson et al., 1990) and in the Peri-Caspian basin (Volozh et al., 1996).

Horizontal extension and shortening play a principal part in the formation of sedimentary basins (McKenzie 1978; Wernicke 1985; Cloetingh and Kooi, 1992). Both these processes affect the evolution of salt structures. The extension of sedimentary layers containing salt results in the thinning of brittle overburdens, faulting, and the activation of salt diapirs (e.g. Jackson and Talbot, 1994). The interplay between the gravitational and buckling instabilities was the subject of our study. We showed that the periodic instability replaces the monotonic instability for rather rapid horizontal stretching or squeezing of the rheologically layered system. We found that this transition can occur at strain rates in the range of $10^{-16}$ to $10^{-15}$ s$^{-1}$ (see Fig. 4), which are reasonable values for the deformations of the overburden in a geological time scale.

Considering the effective viscosity of other basin fills to be greater than the viscosity of salt, we can derive the following geological conclusions from our analytical model.

1. The nature of the gravitational instability in the interface between a layer of buoyant viscous salt and an overlaying layer of strongly non-Newtonian power-law rheology is defined by the behavior of the non-Newtonian material. Salt diapirs are more likely to rise with different wavelengths through a strongly non-Newtonian power-law overburden than they are with a single dominant wavelength.
2. An increase in thickness of a strongly non-Newtonian power-law overburden decreases the dominant wavelength of diapirs rising (upbuilding) through it. This may explain small interdiapir spacings observed in some salt-bearing basins.
3. When the background rate of horizontal extension or shortening is faster than $\sim 10^{-15}$ s$^{-1}$, a buckling instability overwhelms the gravitational instability and the growth rate of diapirs then depends linearly on the effective viscosity ratio.

Thus, analysis of gravitational instability of a salt layer overlain by a strongly non-Newtonian power-law overburden, brings to light two distinct types of deformation: the rheologically layered structure yields viscously for small ratios between the effective viscosity of the overburden and the viscosity of salt, and deforms as a strongly non-Newtonian material when the ratio is higher. Real rocks of course display more complex rheology, but our study of particular special situations represents an essential step in the building of intuition on the behavior of the structure in natural situations.

## Acknowledgements

We are very thankful to B. Birger, H. Koyi and B. Naimark for useful discussions on the subject of the study. We are grateful to A. Braun, W. Jacoby, G. Ranalli and C. Talbot for their constructive reviews and thoughtful comments of an early version of the manuscript. Financial support to A.T.I-Z. from the Royal Society (London), International Science and Technology Center (1293-99), NSF (EAR 9804859) and RFBR (99-05-65050, 99-15-96085) is gratefully acknowledged.

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