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Granular column collapses: further experimental results

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Experimental observations of the collapse of initially static columns of sand in axisymmetric and two-dimensional geometries are presented. The experiments were carried out using cylinders and rectangular tanks 30 to 60 cm tall, and cover aspect ratios between 0.5 and 20, where the aspect ratio is defined as the ratio of the initial height to horizontal extent of the column. The final positions of sand grains from different points initially on the outer surface of the columns are mapped. For all axisymmetric columns the point of maximum runout is found to originate from a point at fractional height 0.74 ± 0.03 of the initial vertical height of the column, independent of the aspect ratio. For two-dimensional columns the corresponding point is 0.65 ± 0.07. Collapses of columns of water-saturated sand into water display a different form of flow, which leads to there being no such well-defined point. In this case, grains from all but the innermost, basal areas of the initial column can end up in the outermost region of the final deposit. For collapses in air and aspect ratios greater than 1, the detail of the initial geometry is relatively insignificant in determining the shape of the final deposit. The results of this and previous studies thus have general applicability, even to situations with less initial symmetry. Movies are available with the online version of the paper.

1. Introduction

Granular media present something of a challenge to conventional physical description. In different situations a granular medium may behave either as a solid (the sand supporting a walker on the beach), a liquid (the same sand poured from a child's bucket) or a gas (the same sand whipped up by the wind into a suspension of particles in the air). Descriptions of granular media using each of these analogies have been partially successful, but do not capture all of the varied behaviour exhibited by such systems. Interesting reviews of the field are presented by Jaeger, Nagel & Behringer (1996), de Gennes (1999), Kadanoff (1999) and Midi (2004).

Materials of this kind may also behave counter-intuitively. When a normal fluid is agitated, it leads to mixing of constituent elements, which is also the thermodynamically expected outcome. However, granular media will often separate into grains of different size, density or composition when shaken (Jaeger *et al.* 1996), a property of clear interest to those attempting to mix, for example, food ingredients, cement, or medicinal products. For these reasons, and also because of the lack of agreed governing equations, theoretical and computational studies of granular materials are difficult to conduct and require experimental verification. In this paper we continue the experimental study of a situation which has received little treatment theoretically: the collapse of an initially static column of grains along a horizontal surface.

Studies of the flow of granular material have motivation in disaster prevention (cliff collapses, landslides and avalanches), industrial production management and specific problems in engineering, agriculture and the Earth sciences. Partly because of the wide range of applications, considerable interest has recently been initiated in this subject. Many studies consider the steady-state situation of a continuous flow down a plane inclined more steeply than the angle of repose; these flows are dominated by frictional effects and often modelled by depth-averaged hydrodynamic equations, as in Savage (1998), Pouliquen & Forterre (2002) and Jop, Forterre & Pouliquen (2006). The aim of the present work is to characterize the behaviour of flows finite in extent and time, dominated by inertial effects. These flows are of current interest in many of the areas described above, for example in the sudden structural failure and consequent collapse of large grain silos, and in the descent and subsequent flow along the ground of material ejected into the atmosphere by volcanic eruption columns (Huppert & Dade 1998).

The work that has already been done on granular column collapses covers axisymmetric and two-dimensional situations and a variety of media (sand, salt, rice, couscous and sugar). One group, working in Bristol and Cambridge, produced a set of quantitative relations for the final runout distance and the time taken to reach this distance as functions of the aspect ratio a, the ratio of the initial height to initial horizontal extent of the column in either axisymmetric or two-dimensional geometries (Huppert *et al.* 2003; Lube *et al.* 2004, 2005). A second group, working independently in Paris on the axisymmetric geometry alone, published very similar quantitative results (Lajeunesse, Mangeney-Castelnau & Vilotte 2004). Both groups also describe the qualitative behaviour of the column in similar terms, as is summarized briefly below. Their results differ only in the assessment of the effect of basal frictional effects, which Lube *et al.* (2004, 2005) believe to be negligible and Lajeunesse *et al.* (2004) believe to influence the final deposit.

The axisymmetric case has also been simulated numerically (Staron & Hinch 2005), by using a simplified model which gives results tuned to be in reasonable agreement with experiment. The simulation uses a completely smooth lower horizontal surface, which, as described below, has little effect on the final deposit, since the main dissipative mechanism is friction with already deposited particles. The scaling laws thus obtained are broadly in agreement with experimental studies. Rather similar calculations were also conducted by Zenit (2005). Additional, two-dimensional experiments have been reported by Siavoshi & Kudrolli (2005), who used an interesting magnetic technique to hold steel beads in the initial position of a step, Balmforth & Kerswell (2005), who emphasized the importance of closely spaced confining walls in the vertical, and Lajeunesse, Monnier & Homsy (2005), who presented velocity profiles of the spreading grains.

A sound theoretical model, with predictions in agreement with experimental data, is still eagerly awaited. Balmforth & Kerswell (2005) explored the results of a shallow-water model, which assumes a uniform flow independent of the height coordinate (not seen in practice) in a layer of small slope (not true for more than half the collapse time). Larrieu, Staron & Hinch (2006) imaginatively patched an initial free-fall regime to a final shallow-water model to suggest power-law dependence of the runout distance with initial aspect in rather moderate agreement with the experimental data, though the multiplicative constant was not well-predicted. Very recently, Jop *et al.* (2006) have boldly suggested rheological equations to govern granular flow by extrapolating results

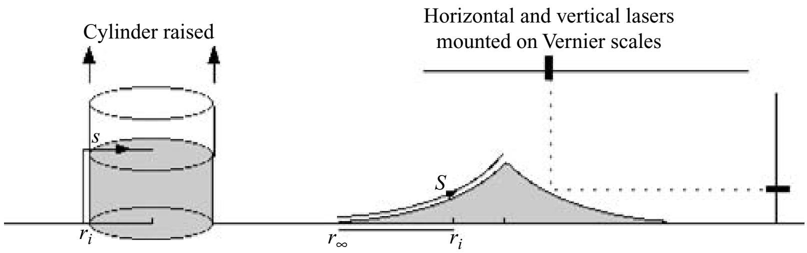

FIGURE 1. (*a*) Set-up of procedure and (*b*) measurement of the final deposit (in the axisymmetric case), including definitions of the arclength coordinates s and S and radial coordinates r_i and r_∞. The two-dimensional case is similar; a single containing wall is raised to release the grains into a rectangular tank and the subsequent wedge-shaped deposit is measured using the same laser equipment.

from flows down inclined chutes. It will be interesting to determine the generality of these equations and their applicability to granular collapses in the future.

The aim of the present work is to investigate further the scalings and geometrical dependence determined by Lube *et al.* (2004, 2005), extending the study to collapses of initially saturated sand into water rather than air. We also discuss the possibility of relating positions in the initial column and the final deposit, the significance of the angle formed at the apex of the final deposit, and, perhaps most importantly, the effect of a less symmetric initial geometry, which determines the applicability of the preceding work to natural and more general situations.

2. Experimental set-up

2.1. *Axisymmetric geometry*

Granular material (dry quartz sand, sorted for grain sizes less than 0.5 mm) was initially contained within one of three cylinders of internal diameters 5.2, 11.4 and 14.6 cm and released quickly by raising the cylinder along a vertical guide. Transparent Perspex cylinders were used, allowing the height of specially placed, horizontal slices of coloured sand to be measured before release. The horizontal surface along which the sand spread was covered with felt, to which sand sticks slightly. With care, this could be used to determine the final resting point of the coloured sand by allowing the top layers to be slid away without disturbing that stuck to the surface. The radii of coloured circles were then measurable with a ruler. The surface used does not alter the characteristics of the flow, since frictional effects at the base have very little effect on the motion of the sand. This is because a thin layer of sand is initially deposited over which the rest of the flow takes place (Lube *et al.* 2005).

The deposit profile was then obtained by using horizontal and vertical Vernier scales with sliding laser pointers to measure the height at given radii for between ten and twenty points on the deposit (depending on the size). Both measurements were accurate to about 1 mm, which represents an error smaller than that observed in repeating the experiment. The experimental set-up, shown in figure 1, closely follows that employed by Lube *et al.* (2004).

2.2. *Two-dimensional geometry*

For the experiments in a two-dimensional geometry, the material was released along a tank of width 10 cm by raising a Perspex containing wall along vertical guides. As before, coloured sand was used to determine the destination of grains from different points within the column. The horizontal surface onto which the sand was released is black tape (easier to fit than the felt), which is smooth and has a considerably lower coefficient of friction than the felt used in the axisymmetric case. As discussed above and by others, the different surface does not affect the flow (Lube *et al.* 2005). As in the axisymmetric case, two Vernier scales and lasers were used to measure the deposit profile. A strip of double-sided sticky tape was stuck to the bottom surface of the tank. This was used to locate coloured sand on the underside of the deposit by carefully allowing the top grains to slide away, leaving visible the bottom layer, stuck to the tape. Then the runout of the coloured stripe could be measured with a ruler.

Two-dimensional collapses of initially water-saturated sand into water were carried out at small aspect ratios using the same set-up and equipment. For larger aspect ratios, a deeper tank had to be used (width 15 cm, depth 60 cm) and the profile measurements were taken from a tracing of the deposit illuminated from one side. The accuracy of this measurement was about 5 mm. In other respects this case was identical to that previously described. Lack of a suitably large tank prevented us from carrying out a collapse of an axisymmetric column into water, which we would expect to display qualitative behaviour similar to the two-dimensional collapses into water and scaling similar to the axisymmetric collapses into air.

3. Experimental observations

3.1. *Description of flows*

Axisymmetric and two-dimensional collapses in air have been previously studied and described by Lube *et al.* (2004, 2005). The flow begins with a spreading phase in which the top surface remains horizontal and falls vertically onto the lower sections, which flow outwards. The grains then flow down the outer surface of the deposit in a layer of movement which gradually becomes thinner as the flow slows. The final phase consists of 'avalanching', where smaller regions of the deposit flow individually. This avalanching phase can cause some modifications to the final deposit (Lube *et al.* 2004). The dynamics of individual avalanches of granular material has been shown to exhibit two distinct types of behaviour corresponding to the collapse of thin and thick layers. The first type propagates downhill from the initial disturbance as sand falling from one part of the slope disturbs lower regions. The second type of avalanche, produced by a greater initial disturbance, propagates in all directions as support is removed from material higher on the slope, which then collapses downward (Daerr & Douady 1999). Both behaviours are observed here, although the thin-layer collapses continue for longer.

The behaviour is found to depend strongly on one non-dimensional geometric factor: the ratio of initial height to horizontal extent (radius) of the column, or the aspect ratio, a. For the axisymmetric situation and aspect ratios less than 1.1, the deposit forms an axisymmetric plateau with a flat top and steep sides. For larger aspect ratios, there is no longer a horizontal top and the deposit is a smooth tapered cone, as is sketched in figure 1(b). In the two-dimensional case, for aspect ratios less than 1.05 the deposit forms a two-dimensional plateau with a rectangular flat area on top and steep sides. For larger aspect ratios, there is no longer a horizontal area on top and the deposit is a smooth, approximately triangular wedge.

The two-dimensional collapse into water happens quite differently. The column slumps, expanding at the base, and then the sand at the leading edge of the column slowly erodes vertically, forming a current which moves out along the base of the tank and deposits sand much further away than it would travel in air. This erosion

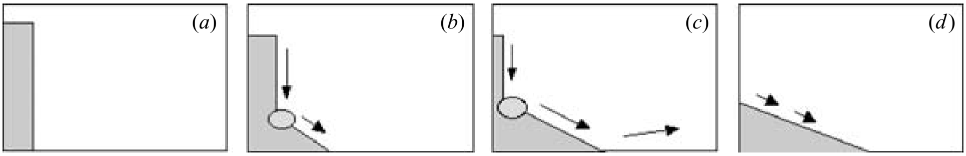

FIGURE 2. A rough sketch of the flow of sand from a two-dimensional saturated column into water, backed up by the photos of figure 3 and the two online movie sequences. A detailed description is provided in the text.

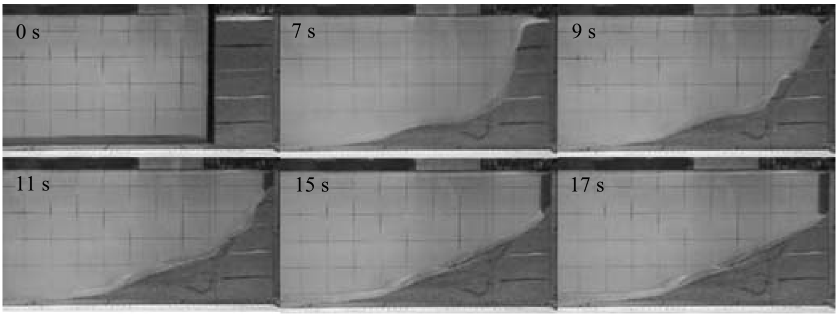

FIGURE 3. Collapse of a water-saturated granular column into water after 7, 9, 11, 15 and 17 s, with coloured grains initially at horizontal levels of 5, 10, 15, 20 and 25 cm.

continues until the edge meets the back wall, and is punctuated by slumping of larger sections of the column onto the slope below. This is sketched in figure 2, where the arrows show the directions of the main flows at each stage. Photos of a typical collapse make up figure 3, and video sequences are available with the online version of the paper as movies 1 and 2. The circles in figure 2 represent areas of turbulence, containing a large amount of suspended material, which occur at the base of the column where falling sand disturbs the bulk of the main deposit. The arrow at the base of the deposit in figure 2(*c*) indicates a flow of material suspended in a gravity current which may extend to several times the horizontal length of the main deposit. This flow represents very little of the volume of the deposit, certainly less than 1 %, and consists of only the smallest grains. Figure 2(*d*) shows the individual collapse of small sections of the deposit in the avalanching phase. Again, both thin- and thick-layer types of avalanche are observed.

3.2. *Destinations of surface grains*

In this section we consider mapping of the positions of grains in the original column to their destinations in the final deposit. This can be done by marking some positions with areas of coloured sand, which are then located afterwards as described in § 2.2. As we shall see, there are problems with this method when the coloured layer becomes too spread out, making it hard first to find the coloured grains and then to decide where to measure their position. We commence our description with the most important, and surprising, result of this subsection: the initial position of the sand which reaches the extremity, $r = r_\infty$.

In the axisymmetric case we find that this sand originated on the surface of the initial cylinder at a height $h_R = Ch_i$, where h_i is the initial height of the column and $C = 0.74 \pm 0.03$ is independent of the aspect ratio, as seen in figure 4. In the two-dimensional case for collapses into air, the corresponding value of C is 0.67 ± 0.07. The scatter is somewhat greater, but there does appear to be a roughly constant

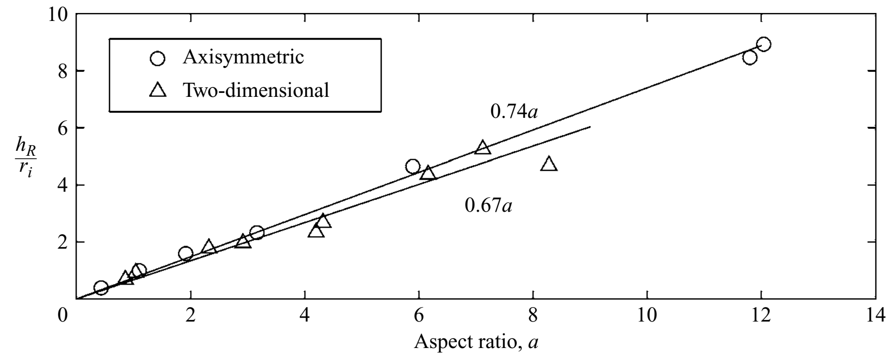

FIGURE 4. h_R/r_i as a function of aspect ratio. The best-fit slope C gives the fractional height of the point of maximum runout in the original column; for axisymmetric collapses this is 0.74 ± 0.03 and for two-dimensional collapses 0.67 ± 0.07.

relationship for aspect ratios less than 10. For aspect ratios greater than 10 it is difficult to carry out the evaluation with enough accuracy to form any conclusions, since the coloured sand used to map points becomes more widely distributed. For the two-dimensional collapses into water, it was not possible to determine the origin of the point of furthest runout because the furthest points on the final deposit are made up of sand from all regions of the outer (vertical) edge of the column.

We arrive at these conclusions and additionally determine the initial point of each grain of sand on the surface of the resultant deposit by defining s to be the arclength extending along the outer surface of the initial column starting at the base and continuing at $h = h_i$ to the centre $r = 0$ (see figure 1a) and S the arclength along the same surface in the final deposit (starting from the same point on the base, a distance r_i from the centre; see figure 1b). The parameters s and S uniquely (to within the symmetry of the geometry) define a point on the outer surfaces of the initial column and the final deposit respectively.

In the axisymmetric case, plotting the normalized S/S_{max}, where S_{max} is the maximum value S, against s/s_{max}, where s_{max} is the maximum value of s, results in an inverted sigmoid curve as seen in figure 5(a). This form of curve is not observed in the two-dimensional case.

It has been suggested by Lube *et al.* (2004) that all grains initially on the surface remain on the surface and no grains from within the column end up on the surface of the deposit, by analogy with the equivalent fluid mechanical property. For axisymmetric collapses we find this to be correct. However, for two-dimensional collapses in either air or water, this does not necessarily hold. For two-dimensional collapses in air, the curves, as displayed in figure 5(b), do not in general reach $S/S_{max} = 1$, unlike the axisymmetric case where all do. This difference is because the sand initially on the top of the two-dimensional column slips sideways and does not end up at the very top of the deposit formed. The top of the deposit consists of grains which began the collapse within the centre of the column.

For two-dimensional collapses in water, the different dynamics of the collapse, as described earlier, lead to a very different situation; most importantly, the mapping from s to S is no longer continuous. There are some final stripes visible on the surface of the deposit, which are in the opposite order to their appearance in the initial column (see figure 3). The areas of the final deposit that are far from the back wall consist of mixed sand from all the outermost regions of the column. For this

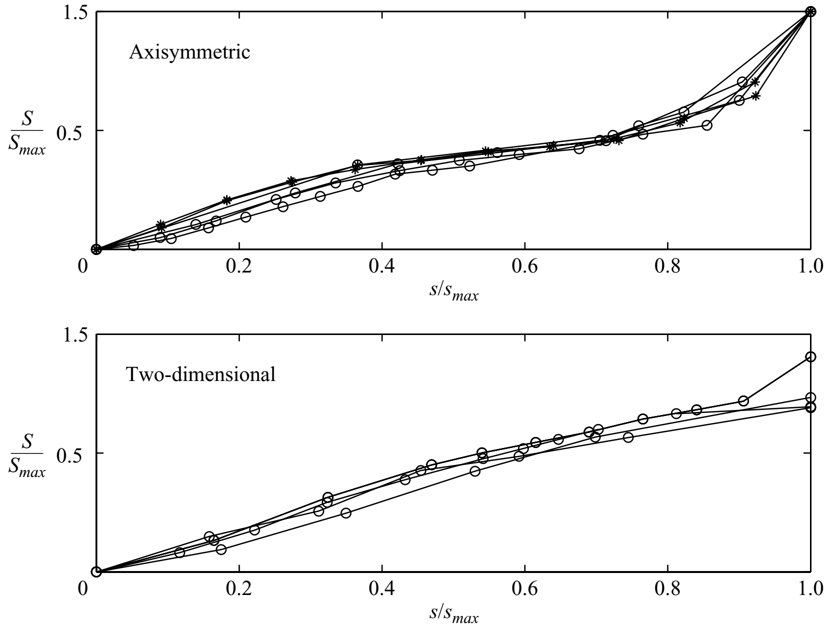

FIGURE 5. Fractional distances along the outer surfaces of initial and final columns: S/S_{max} as a function of s/s_{max} in axisymmetric and two-dimensional geometries, for varying initial heights and radii.

reason, the curve $S(s)$ has an increasing section at the start, is undefined in the middle, and has a decreasing section at the end. The lack of good repeatability of each experiment and the widening of coloured regions (due to the turbulent region, which mixes coloured and plain sand) make it difficult to determine the shape of even the well-defined parts of the curve more accurately.

3.3. *Angle at the apex of the deposit*

The angle at the apex of a deposit formed by the collapse of a granular column may differ from the static angle of repose for that material, but this has not previously been systematically investigated. The profiles described above were used to estimate the angle formed at the top of the cone (or at the edge of the plateau), by a straight or quadratic fit through the last four points, as seemed appropriate for the data. Errors are introduced in trying to determine an angle from a set of points; a straight or quadratic fit was used according to which appeared more appropriate by eye, but clearly in both cases a compromise has to be made between the possible inaccuracy of the two final points and the wish to find the angle as close as possible to the apex. However, it seems likely that more of the scatter is due to the lack of repeatability of the experiment than to the details of this determination. There was clear variability even on different regions of the same deposit. The results are shown in figure 6 and display at least 5° of scatter for large aspect ratios. It has been observed previously by Lube *et al.* (2005) that the final avalanching stage of the collapse may introduce much of the variability between experiments; this would presumably also have greatest effect at the point of greatest slope, i.e. close to the top of the cone or to the edge of the plateau.

For aspect ratios less than 3, there is a large variability in the calculated angle and little pattern in the data other than that the axisymmetric deposits appear to have

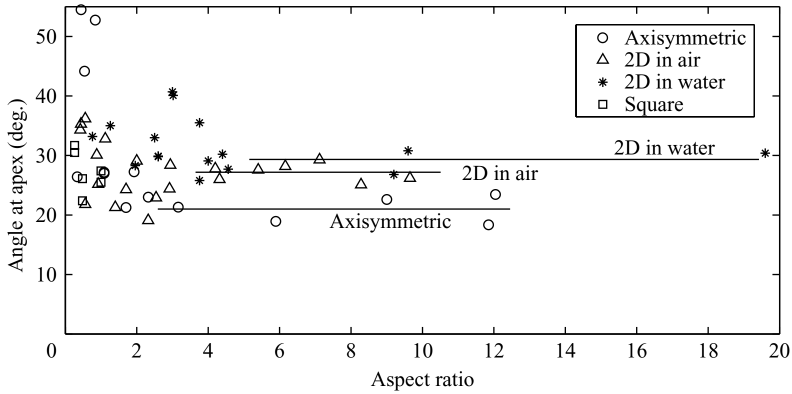

FIGURE 6. Angle at the top of the deposit for axisymmetric (in air) and two-dimensional (in air and water) cases. Some data are also given for experiments on columns of square cross-section, discussed in § 3.5.

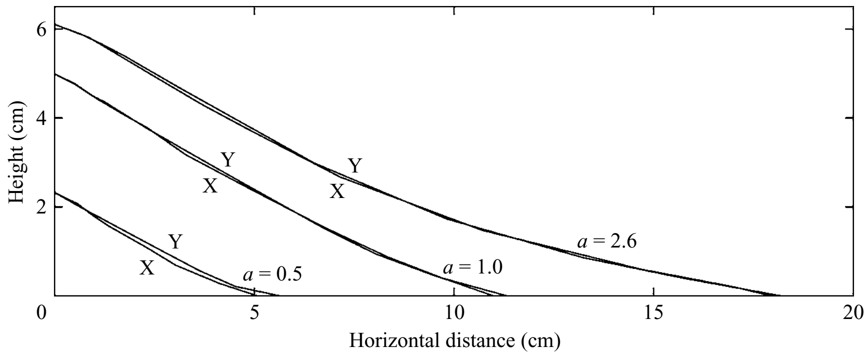

FIGURE 7. Cross-sections of the final deposit through the face (X) and corner (Y) of a collapsed column with square base, for aspect ratios 0.5, 1.0 and 2.6.

the steepest-sided plateaux at low aspect ratio. It is here, perhaps owing to the slower nature of the collapse, that the final angle may exceed the angle of repose for sand, which is approximately 30°.

For aspect ratios greater than 3, there is a more obvious pattern to the results: axisymmetric deposits have the smallest (least steep) angle at the top of the deposit, about 21°; the angle formed by two-dimensional collapses into air is about 27°; and that of two-dimensional collapses into water is about 29°, with an experimental error of about 3° for each value. These differences may be due to geometric factors (the back wall and the sand to each side support grains in the two-dimensional case) and the extra pressure of the water also providing support.

3.4. *Square geometry*

Axisymmetric and two-dimensional collapses show variation only in one dimension along a radius from the central point or line. Natural situations are unlikely to show such high symmetry. Thus it is useful to have an indication of the effects of a differently shaped column.

In order to determine this, collapses with columns of square cross-section were performed and the profiles measured along two radii (through one corner and through the midpoint of one side, as in figure 7). For aspect ratios less than about 0.8 (where a plateau was left on the top of the deposit), the section through the side resembled

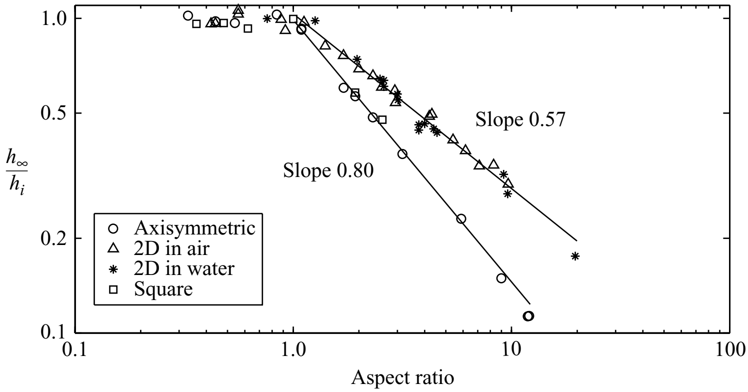

FIGURE 8. Ratio of final height at the centre to the initial height as a function of the aspect ratio for axisymmetric, two-dimensional and square cases. The two-dimensional situations in air and water give the same scaling, and the square geometry is similar to the axisymmetric case.

a two-dimensional profile and through the corner an axisymmetric profile. It was found that for aspect ratios greater than approximately 1 the final deposit was indistinguishable from that of an initially axisymmetric column of equal aspect ratio and volume. Such collapses also follow the scaling laws for axisymmetric collapses as discussed briefly in the next subsection.

This result indicates that conclusions from studies of high-symmetry cases should require only little modification for application to other, more naturally occurring situations.

3.5. *Final height*

It was previously proposed by Lube *et al.* (2004) that, for the axisymmetric case, the ratio of final height to initial height scales as $a^{5/6}$. The results presented here (figure 8) give an exponent of 0.80 ± 0.04, consistent with a value of $5/6 = 0.83$. The two-dimensional case clearly follows a different scaling law; Lube *et al.* propose a value of $3/5 = 0.60$, which is within the error margin of the best-fit determined here of 0.57 ± 0.04. This is found to be the same for two-dimensional collapses in air and in water, despite the rather different motions involved, which further demonstrates that viscous effects are negligible in the main body of the flow.

Some results for a square geometry are also shown. These confirm our previous remark that the collapse becomes essentially indistinguishable from an axisymmetric collapse for aspect ratio greater than 1.

4. Summary and discussion

New details of axisymmetric and two-dimensional collapses into air have been determined. The origins and destinations of surface grains with respect to a non-dimensional measure of arclength along the outer surface were found. The resulting graphs were used to find the initial height of the point of maximum runout in the final deposit. For collapses into air, this was calculated as 0.74 ± 0.03 and 0.67 ± 0.07 times the full initial height, independent of the aspect ratio, for axisymmetric and two-dimensional situations respectively. For collapses into water, no such point existed owing to mixing of layers from different regions of the initial column.

Two-dimensional collapses of initially saturated sand into water show a different sequence of events during the collapse, but the scaling of the final height and final runout distance with aspect ratio is virtually the same as for dry sand in air. This demonstrates that the bulk properties (although not the details) of the flow are dominated by inertia rather than intergranular friction (the effective 'viscosity').

We find for collapses in air that the detail of the initial geometry is relatively insignificant; the deposit resulting from the release of a square column of aspect ratio greater than 1 is difficult to distinguish from that resulting from an axisymmetric column of the same aspect ratio and initial volume of sand. Thus it seems likely that the results of this and previous studies have general applicability even to situations with less symmetry, i.e. to the naturally occurring cases described in § 1. A possible next step would be to collate some real observational data on these situations and assess the effectiveness of this model.

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