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Initial advance of long lava flows in open channels

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ABSTRACT

The initial development of long lava flows is investigated using simple theory and field evidence. Order-of-magnitude estimates of the evolving thickness and the extending length of lava are obtained by scaling arguments based on the simplification that the bulk structure can be modelled initially as a Newtonian fluid. A scaling analysis suggests that the rate of advance of the leading front evolves primarily due to temporal variations in the effusion rate and minimally due to topography. The apparent viscosity of the bulk flow increases with time at subsequent stages when effects due to cooling become important. Theoretical results are applied to the study of long lava flows that descended on Etna, Kilauea and Lonquimay volcanoes. We determine that lava flows at Kilauea extended initially like a Newtonian fluid with constant viscosity, implying that thermal effects did not significantly influence the dynamic properties of the bulk flow. In contrast, effects due to cooling played a major role throughout the advance of lava flows at Etna and Lonquimay. We show that the increasing length and volume of an active emplacement field can be monitored to estimate its evolving viscosity, which in turn allows the further advance of the lava to be predicted.

1. Introduction

Lava flows occur when molten rock is extruded from a volcanic vent. A long channel of lava may develop down a slope, where the flow is driven by gravity and confined laterally by elevations in topography. The lava quickly cools and solidifies at the margins of the flow, where levees form and further confine the flow (Hulme, 1974; Sparks et al., 1976). The upper surface of the lava may also solidify to form a lava tube system whereby the lava continues to flow inside a completely enclosed passage (Greeley and Hyde, 1972; Hallworth et al., 1987; Calvari and Pinkerton, 1999). The confinement may insulate the interior of the channel, allowing the lava to flow efficiently, without much loss of heat, towards its leading front (Keszthelyi, 1995). The front of the flow may propagate, branch into different lobes and stagnate in a complex series of processes (e.g., Lipman and Banks, 1987), before the entire structure of the emplaced lava solidifies.

One of the motivations for understanding the morphology of lava flows is to predict and evaluate the consequences of an effusive eruption. The resultant flow of hot and destructive lava can reach distant areas, threatening lives and damaging properties (Blong, 1984). An accurate prediction of the evolution of active lava flows is helpful for identifying danger zones and assessing risks posed to areas on volcanoes. For the purposes of effective forecasting, it is useful to be able to predict the extent of lava based on conditions that can be measured prior to or during the early stages of lava emplacement.

Previous studies obtained empirical relationships showing that the final lava length is correlated to a number of factors, including the mean effusion rate at the vent (Walker, 1973), the total erupted volume (Malin, 1980) and the rheology of the lava (Pinkerton and Wilson, 1994). An idea has been put forward that the flow is either cooling-limited when it reaches a maximum length attainable for a given supply of lava from the vent or volume-limited when a considerable decline in the effusion rate prevents the flow front from reaching the maximum length (Guest et al., 1987). In either case, the final length of the lava is controlled by dynamic processes involving heat loss and depends importantly on the effusion rate (Harris and Rowland, 2009). We complement previous studies by examining dynamically how the various input factors, including variations in the effusion rate and effects due to cooling, influence the lava flow before it ultimately stops.

The dynamic processes leading to the final solidified state require understanding of the fluid dynamics of the lava (Griffiths, 2000). Of particular importance is the development of lava flows during their early stages, when the flow front advances rapidly and reaches a large proportion of its final extent. A quantitative formulation of the initial advance of long lava flows forms an important foundation for studying subsequent stages of the evolving morphology of lava. The use of scaling arguments, which are applied to the early stages, could be developed further to study other problems including the prediction of the final lava extent, which is beyond the scope of the current investigation.

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During the early stages, an open channel develops down a slope and directs the flow towards its advancing front (Hulme, 1974; Kerr et al., 2006). The resultant flow during the early stages is commonly modelled as Newtonian and laminar (Tallarico and Dragoni, 1999; Sakimoto and Gregg, 2001). There is a rich class of mathematical problems relevant to the prediction of lava flows (Baloga and Pieri, 1986; Bruno et al., 1996). The apparent viscosity of the bulk structure of the lava is expected to remain constant until the flow is influenced by thermal effects, which change the dynamical properties of the lava in two important ways. First, crystals may nucleate due to degassing and grow in the flow, effectively increasing the viscosity of the lava (Sparks et al., 2000). Second, a crustal layer may develop on the surface due to cooling (Griffiths and Fink, 1993), effectively introducing an additional resistance to flow. Both mechanisms reduce the flow speed considerably until the flow stagnates altogether.

Here, we consider the initial advance of lava flows supplied down open channels of different shapes. The aim is to provide theoretical insight into natural lava flows by simplifying the analysis as much as possible while including the most fundamental mechanisms. For simplicity, the lava is modelled as Newtonian and we study the effects on the bulk flow due to given variations in the topography, the effusion rate and the apparent viscosity. Theoretical results obtained are applied to describing lava flows that descended the volcanic slopes of Etna, Kilauea and Lonquimay. The theoretical treatment is presented first in Section 2, followed by applications to field data in Section 3. We demonstrate how increases in the viscosity of the lava and further advance of the flow front can be predicted solely based on prior measurements of the cumulative volume and the length of an evolving emplacement field.

2. Theory

Consider an open channel of lava flowing down a slope. The channel may represent topography confining the entire length of lava that has been extruded from a volcanic vent, as long as the flow does not split into different branches. The following analysis applies equally well to lava that has branched off from another channel and extends thereafter as a single lobe. We are concerned with the temporal evolution of the dimensions of a single channel of lava. Of interest are the characteristic height $H(t)$, width $W(t)$ and length $L(t)$ of the flow at time t. The flow is primarily along the channel, provided that the dimensions of the flow satisfy $H \ll W \ll L$.

The exact shape of the channel confining the flow of lava will depend on a number of factors, which include the pre-existing topography and the development of levees at the margins of the flow. However, as we discuss later, the details of the channel do not significantly influence the flow. Consider a general relationship between the width and thickness of the flow of the form

$$H / w_{*} \sim (W / w_{*})^{n}, \tag{1}$$

where ~ denotes a relationship of proportionality, w is a measure of the size of the channel and n is a prescribed constant that describes the shape of the channel. For example, the limit as $n \to \infty$ is equivalent to $W \sim w$ and corresponds to a shallow layer of lava flowing down a flat channel of constant width w. Channels confining lava are approximately described by n taking some finite value greater than 1. For instance, $n = 2$ corresponds to a thin layer of lava flowing inside a channel of cylindrical shape whose radius of curvature, w, is much larger than the characteristic flow thickness, as shown in Fig. 1. The case of $n = 1$ describes flow along a wedge. Deep and narrow flows, described by $n < 1$, are not considered here because they are sheared predominantly across fractures of width $W \ll H$ and do not apply to natural lava flows.

The exact velocity varies within the lava but has a common characteristic magnitude denoted by $U(t) \sim L/t$ because L is the only characteristic length scale associated with the direction of flow along the channel. In particular, U is the characteristic rate of advance of the flow front, which is estimated by considering the governing equation of Newtonian and laminar flow. The driving force of gravity must balance the resistive forces due to the viscosity of the lava, provided that inertial effects are negligible. The component of gravity in the direction of the flow is given by $\rho g sin\theta$, where ρ is the density of the lava, g the acceleration due to gravity and θ the angle of inclination of the channel to the horizontal. The lava is sheared predominantly across its thickness because the resistive forces exerted at the sides of the flow are negligible, since $H \ll W$. The flow is sheared at the base, where we impose the condition of no slip. Shear stresses exerted by the ambient or any development of a crustal layer on the free surface are considered to be small initially. Given that the flow is sheared across its thickness, the viscous forces scale like $\mu U / H^2$, where μ is the dynamic viscosity of the lava. By balancing gravity with viscous forces and rearranging, we obtain the characteristic speed of the flow

$$U \sim \rho g \sin\theta H^{2} / \mu. \tag{2}$$

Note that Eq. (2) is consistent with an equation quantifying the surface velocity of flow down a channel with rectangular cross-section, often referred to as the Jeffreys equation (Jeffreys, 1925). The flow speed depends importantly on H, the thickness of the flow, which is set by the supply of lava into the channel.

The supply of the lava into the channel from upstream depends on the effusion rate at the vent and generally varies with time. The effusion rate corresponds to the rate of change of the cumulative volume of extruded lava. Typically, the effusion rate increases initially during a waxing phase and then decreases slowly during a waning phase (Wadge, 1981). To illustrate the effects of the lava supply on the resultant flows down open channels, we consider a simple power-law dependence of the effusion rate with time, which is expected to fit field data during the initial stages of an effusive eruption. Let the effusive activity at the vent begin at time $t = 0$ such that the cumulative volume of extruded lava is given by

$$V = V_{*}(t / T_{*})^{\alpha}, \tag{3}$$

where T is some fixed time scale at which the volume erupted is $V = V$. The exponent $\alpha \geq 0$ is a prescribed constant and describes the temporal evolution of the effusion rate at the vent. For example, $\alpha = 0$ corresponds to a fixed volume V of lava extruded rapidly at time $t = 0$ and no further extrusion subsequently. Another example of importance is $\alpha = 1$, which corresponds to a continuous supply of lava with a steady effusion rate at the vent. A more general situation, where the effusion rate at the vent declines continuously with time, is described by α taking some value between 0 and 1. Note that the volume of extruded lava for general $\alpha > 0$ in Eq. (3) grows indefinitely, which does not model the effusion rate at large times. Nevertheless, Eq. (3) provides useful insight into the development of long lava flows during the early stages of the propagation, as we investigate below.

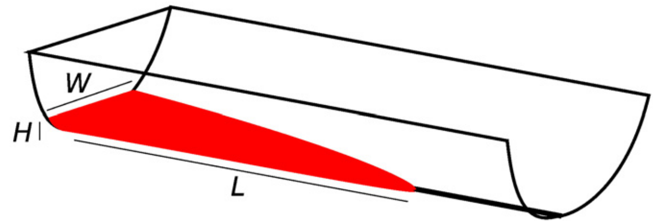

Fig. 1. A sketch of lava of typical length L, width W and height H flowing inside a channel of cylindrical shape.

The cumulative volume of lava in the channel given by Eq. (3) must scale like

$$V \sim HWL. \tag{4}$$

By eliminating W, H and V from the coupled relationships in Eqs. (1)–(4) and the relationship $U \sim L/t$, the scaling for the extent of the flow at time t is given by

$$L \sim t^{(2\alpha n + n + 1)/(3n + 1)}. \tag{5}$$

The exponent $c = (2\alpha n + n + 1)/(3n + 1)$ depends only on the evolving nature of the effusion rate at the vent described by α in Eq. (3) and the shape of the channel described by n in Eq. (1). The exponent c does not depend on the viscosity of lava or the slope angle of the channel, as long as they remain approximately constant.

The scaling of the flow length given by Eq. (5) is consistent with a more detailed theoretical analysis and agrees well with data from a series of laboratory experiments (Takagi and Huppert, 2007; Takagi and Huppert, 2008). The experiments were conducted by supplying glycerin inside a wedge or a semi-circular channel either instantaneously or continuously with a steady flux, verifying the validity of Eq. (5) when $n = 1, 2$ and $\alpha = 0, 1$. The special case of $\alpha = 0$ not only describes the flow of an idealized situation of an instantaneous release of fluid into an inclined channel, it also describes the flow of a more general situation where the fluid is supplied from the upstream end of an inclined channel over a duration that is much shorter than the time scale of the resultant flow. In the context of a brief effusive eruption, where a considerable volume of lava is extruded during a relatively short period, the resultant flow does not depend on the details of the effusive activity and extends like Eq. (5), with $\alpha = 0$.

The numerical value of the exponent c, previously introduced so that the flow extends with time like t^c, provides an important insight into the evolving nature of lava flows in open channels. Depending on whether $c < 1$, $c = 1$ or $c > 1$, the position of the front decelerates, progresses steadily or accelerates, respectively, with time. A close inspection of the dependence of the exponent c on n in Eq. (1) and α in Eq. (3) reveals that the shape of the topography is not as important as the nature of the temporal variations in the effusion rate. This is illustrated by a stronger dependence of c on α than n, as shown in Fig. 2 for $\alpha = 0$, 1 and 2. In fact, in the special case of a steady effusion rate at the vent ($\alpha = 1$), the flow front advances steadily in open channels of any shape.

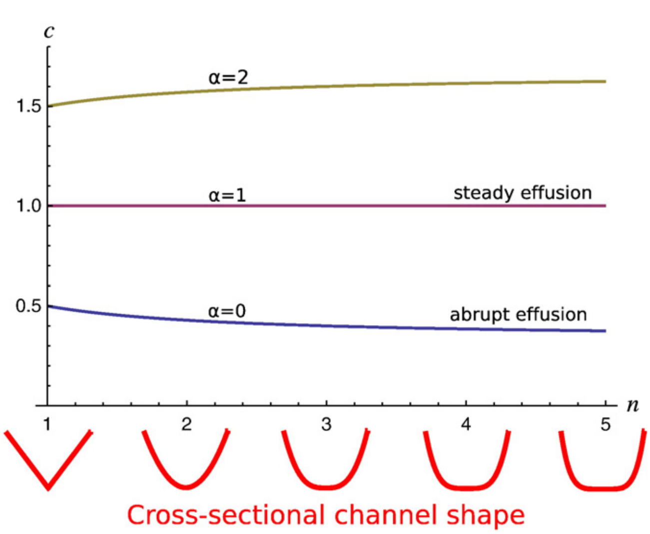

Fig. 2. Plot of c against n for three different values of α, where a cumulative volume $\sim t^{\alpha}$ results in a lava flow of length $\sim t^c$. The number n describes the cross sectional shape of the channel, as sketched beneath corresponding integer values of n.

Given that the shape of the channel does not significantly influence the exponent c, a representative shape of a channel with rectangular cross-section will be considered in all the analysis to follow. The flow is effectively two-dimensional, independent of the cross-stream direction. By taking the limit as $n \to \infty$ in Eq. (5), we determine the extent of the flow down a flat channel of characteristic width a to be given by

$$L \sim \left(\rho g \sin\theta A_*^2 T_*/\mu\right)^{1/3} (t/T_*)^{(2\alpha+1)/3}, \tag{6}$$

where $A\ = V\ /w$ is the characteristic area of lava extruded in time T per unit cross-stream width w. The extent of the flow advances at least like $t^{1/3}$, provided that the lava descends down the channel like a Newtonian fluid of constant viscosity. Note that L in Eq. (6) does not change much as a result of minor variations in θ and w, reinforcing the idea that topographic variations play a minor role in governing the length scale of the flow. By coupling relationships in Eqs. (3) and (6), we determine the characteristic thickness of the flow to be given by

$$H \sim [\mu A_*/(\rho g \sin\theta T_*)]^{1/3} (t/T_*)^{(\alpha-1)/3}. \tag{7}$$

A declining supply of lava into the channel with $\alpha < 1$ in Eq. (3) leads to a decreasing thickness of the lava with time, as expected. The scalings in Eqs. (6) and (7) are consistent with the mathematical solutions obtained by a more detailed analysis (Lister, 1992).

When the length and thickness of lava flowing down an inclined channel scale like Eqs. (6) and (7) respectively for some common value α, the implication is that the lava is flowing like a Newtonian fluid of constant viscosity. If the temporal variations of either Eq. (6) or Eq. (7) disagree with α in Eq. (3), which can be measured independently by monitoring the cumulative volume of lava, then the theoretical assumptions made so far need to be revised. The obvious source of the problem lies in the simplifying assumption so far that the viscosity remains constant. As the lava cools, its viscosity may increase by orders of magnitude.

It is helpful to consider the effects on the flow due to a time-dependent viscosity of the form $\mu \sim t^{\beta}$ for some β. Although this power-law is not the end result of an explicit theoretical development, a similar analysis has provided insight into the growth of lava domes (Sakimoto and Zuber, 1995). The specific viscosity of lava may vary in position but the bulk structure is simplified to flow with an apparent viscosity, whose order of magnitude depends only on time. By setting $\mu \sim t^{\beta}$ in Eq. (6), the characteristic thickness of the flow evolves in the form

$$H \sim t^{(\beta+\alpha-1)/3} \tag{8}$$

behind an advancing front of the flow, extending a distance

$$L \sim t^{(2\alpha+1-\beta)/3}. \tag{9}$$

Relationships Eqs. (8) and (9) indicate that an increase in the viscosity with time, represented by $\beta > 0$, results in a channel of thicker lava, which advances at a slower rate compared to the corresponding flow without any increase in viscosity. For example, a significant rise with time in the level of lava inside a channel, which is being supplied at a steady rate, can be explained by setting $\alpha = 1$ in Eq. (8) and deducing that the viscosity of the lava is increasing considerably with time. When the characteristic viscosity $\mu \sim t^{\beta}$ increases with time such that $\beta \geq 1 + 2\alpha$, the scaling given by Eq. (9) indicates that the flow front does not advance, but that the flow extends a finite distance and stops. Given that natural lava flows do not continue to propagate indefinitely, the implication is that the viscosity of the lava must increase sharply with time before the flow stops.

Another possibility that gives rise to a considerable decline in the speed of the flow is the additional force resisting the motion due to the formation of a crustal layer on the upper surface of the flowing lava. Effects on the lava flow due to the development of a surface crust are presented below for the propagation down a slope, which is different from a growing lava dome on a horizontal surface (Griffiths and Fink, 1993). The idea is that cooling is assumed to be confined to a thin thermal boundary layer of characteristic thickness δ near the crust, below which the interior flow of lava remains isothermal. The approximation holds for a sufficiently large Peclet number $UH/\kappa \gg 1$, where κ is the thermal diffusivity of the lava flowing under the crust. Thermal conduction is assumed to be most significant across the depth so that the crustal thickness grows diffusively like

$$\delta \sim (\kappa t)^{1/2}. \tag{10}$$

The crust is approximated to have a thickness proportional to the thermal boundary layer and exert a shear stress $\sigma_c \delta$ on the flowing lava per unit surface area, where σ_c is the effective shearing strength. Under these assumptions, the driving force due to gravity is no longer balanced by viscous forces at the base of the channel and within the flowing lava but instead by $\sigma_c \delta / H$, the retarding force per unit volume due to the surface crust. By setting $\rho g sin\theta \sim \sigma_c \delta / H$ and coupling with Eqs. (3) and (4), we determine the characteristic thickness of the bulk flow to grow like $t^{1/2}$ behind an advancing front, which extends a distance

$$L \sim t^{\alpha - 1/2}. \tag{11}$$

Note that Eq. (11) is equivalent to Eq. (9) with $\beta = 5/2 - \alpha$, suggesting that the bulk flow with a surface crust extends like a fluid with apparent viscosity that grows in time like $t^{5/2-\alpha}$. Note also that an advancing flow front requires a continuous supply of lava with $\alpha > 1/2$. The scaling given by Eq. (11) does not hold for a cumulative volume of lava described by $\alpha \leq 1/2$, suggesting that the entire structure of the flow cools and comes to rest when there is insufficient supply of lava driving the flow in the insulated interior under the crust.

The applicability of the scaling laws introduced in this section is best assessed by testing against data of natural lava flows. The theory predicts the characteristic thickness of lava to scale like Eq. (8) and the extending length like t^c as in Eq. (9), provided that the cumulative volume scales like t^α and the apparent viscosity of the lava like t^β. The scaling relationships are useful for estimating any variation with time in the viscosity of the lava, which is difficult to measure directly in the field. When the cumulative volume of lava is plotted against time on logarithmic scales, the slope of the line of best fit is α. Similarly, c can be obtained by finding the slope of the line of best fit through data of the flow extent plotted against time on logarithmic scales. The theory predicts the apparent viscosity to have scaled like t^β, where $\beta = 1 + 2\alpha - 3c$. In the following section, we demonstrate how the evolving viscosity and the morphology of natural lava flows can be explained using simple ideas that have been developed. It is not possible at this stage to conduct a complete test of the model partly due to the lack of data showing changes in the bulk viscosity of the flowing lava. Variations in the viscosity are inferred and remain to be tested against further data in future studies.

3. Natural lava flows

Many natural lava flows have been observed and studied extensively. Well-documented sets of field data are available in the literature describing the evolving morphology of long lava flows. We select representative sets of data to examine how a single channel of lava advances down a slope. The width of the flow and the slope do not change considerably with time or distance downstream, as assumed in the model. Lengths of the flow recorded at different times were compiled to investigate several different lava flows, as presented in Table 1. The selected data provide us with an excellent opportunity to test the theoretical ideas developed in Section 2.

Table 1
Table of selected lava flows investigated in this article.

Location	Start date	Reference
Etna	18 Jul 2001	LFS1, Coltelli et al. (2007)
Kilauea	13 Jun 1983	Episode 4, Wolfe et al. (1988)
Kilauea	22 Jul 1983	Episode 6, Wolfe et al. (1988)
Kilauea	30 Mar 1984	Episode 17, Wolfe et al. (1988)
Lonquimay	27 Dec 1988	Naranjo et al. (1992)

A series of eruptive events occurred on the Kilauea Volcano of Hawaii starting in 1983. Episodes of vigorous fountaining at central vents resulted in basaltic lava flows, some of which extended several kilometers. Detailed narratives and graphical representations of lava flows during the first 20 episodes in 1983–1984 are available in (Wolfe et al., 1988). We take a representative sample of episodes in which one major flow of lava extended from the central vent of Pu'u O'o.

A cone of Pu'u O'o marked the locus of major fountaining and lava discharge soon after the start of episode 4 during the late morning of 13 June 1983. The cone enclosed a crater partly filled with lava, which overflowed and fed a well-developed channel. The lava flow advanced in the southeastern direction and extended a distance of 5.7 km by 16:00 on 15 June 1983. During this time, a time-lapse camera at the vent recorded low bursts of fragmented spatter rather than sustained fountains, which caused large fluctuations in velocity and thickness of the major flow. However, variations in fountain height in episode 4, as plotted in figure 1.24 of Wolfe et al. (1988), are not significant over the time scale of tens of hours during which the flow front advanced. Based on the assumption that fountain height is proportional to discharge rate of lava at the vent, the major channel was fed with lava at an approximately steady rate. The advancing position of the flow front is plotted against time in Fig. 3. Although the flow is reported to have experienced fluctuations in speed, the general trend is an approximately steady advance of the flow front until the afternoon of 16 June 1983. Both the discharge rate at the vent and the rate of advance of the major flow were approximately steady during the course of about three days so $\alpha = 1$ and $c = 1$. We deduce $\beta = 1 + 2\alpha - 3c = 0$, meaning that the major channel of lava flowed initially like a Newtonian fluid of constant viscosity.

Some subsequent episodes also featured a steady discharge rate of lava at the Pu'u O'o vent which resulted in a steady advance of a major flow. In episode 6, after a series of minor lava flows, one major channel of lava developed and extended to the north east. Fig. 3 shows how

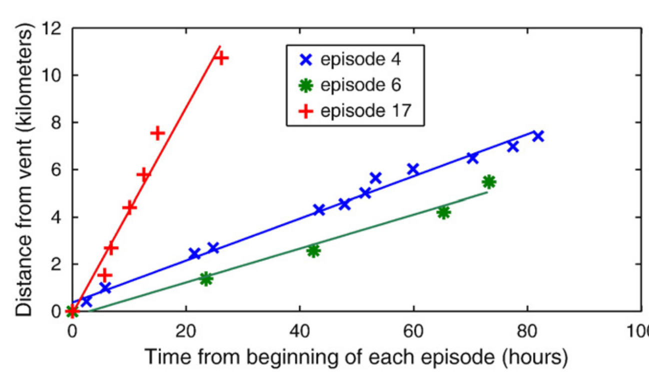

Fig. 3. Extent of the advancing front of lava plotted against time during three representative episodes of the Pu'u O'o eruption in Kilauea, 1983–1984. Data points are reproduced from figures presented on the plates of (Wolfe et al., 1988).

the length of the flow initially extended. The rate of advance of the major flow remained steady from the early afternoon of 23 July 1983, when the channel began to extend, until the night of 24–25 July 1983, when the flow front divided into two parallel lobes. During this period, a steady and sustained discharge of lava supplied the major channel, while the flow front advanced at an average rate of 80 m/h, suggesting that the viscosity remained constant.

In episode 17, a major channel of lava extended eastwards at an approximately steady rate of 490 m/h from a distance 2 km away from the base of Pu'u O'o, from the late morning until the evening of 30 March 1984. During the day, the fountain height remained at approximately 100 m, suggesting that the discharge rate of lava was steady. The discharged lava was supplied primarily to the main eastern flow and minimally to its minor subordinate flow, as shown graphically on the map of episode 17 on plate 2 of Wolfe et al. (1988). The considerably faster flow in episode 17 compared to other episodes on Kilauea has been attributed partly to confinement of the flow (Wolfe et al., 1988). However, the average width of the flow was 220 m in episode 17, not much less than the widths of 230 m and 260 m in episodes 4 and 6 respectively. An increase of the effusion rate by a factor of almost four and a minor decrease in the flow width, which would increase $A^{2/3}$ in Eq. (6) by approximately 2.5, do not account for the increase in the flow speed by a factor of approximately six. The fast flow in episode 17 is primarily due to an order-of-magnitude decrease in the viscosity of the lava. This is consistent with Wolfe et al. (1988) who argue that increasing lava temperature, decreasing phenocryst content and changing lava composition may have been related to this apparent change in viscosity compared to earlier episodes.

On 27 December 1988, a major flow of andesite lava developed on Lonquimay Volcano in Chile which can be studied as follows. The main flow moved north-northeast down the Rio Lolco valley and extended to 10.2 km after 330 days. It has been observed that the lava of thickness ~20m and width ~500m extended in length ~10^4m within months by order of magnitude. Using these values in the scaling relationship in Eq. (4), we obtain an order-of-magnitude estimate of the cumulative volume of extruded lava ~$10^8 m^3$, which is consistent with the measurement of the final volume of extruded lava, $2.3\times10^8 m^3$ (Naranjo et al., 1992). The position and thickness of the continuously advancing flow front at different stages are presented in table 3 of Naranjo et al. (1992). A plot of the flow length against time on logarithmic scales is shown in Fig. 4. By calculating the slopes of the lines of best fit, we determine that the flow extended like $t^{0.45}$ for approximately the first 8 days and like $t^{0.18}$ thereafter, before the flow stopped. The cumulative volume of the flow increased approximately linearly with time at a rate of $6.9\times10^6 m^3$ per day for the first 8 days according to Fig. 3 of Naranjo et al. (1992). Given that the cumulative volume scaled like t and the flow extent like $t^{0.45}$, the theory developed previously predicts that the apparent viscosity increased with time like $t^{1.65}$ initially. Eq. (11) with $\alpha=1$ predicts $c=0.5$, which is in reasonable agreement with the field data indicating $c=0.45$, suggesting that the initial lava flow on Lonquimay was resisted by the development of a surface crust. At subsequent stages, the cumulative volume increased like $t^{0.34}$, where the exponent corresponds to the slope of the line of best fit through the corresponding data in Fig. 4. Setting $\alpha=0.34$ and $c=0.18$, we obtain $\beta=1.14$ and deduce that the apparent viscosity increased with time like $t^{1.14}$. The theory shows that the viscosity of the lava increased with time throughout the course of the flow, which is consistent with the increase in the viscosity of the flow front as it advanced downstream (Naranjo et al., 1992).

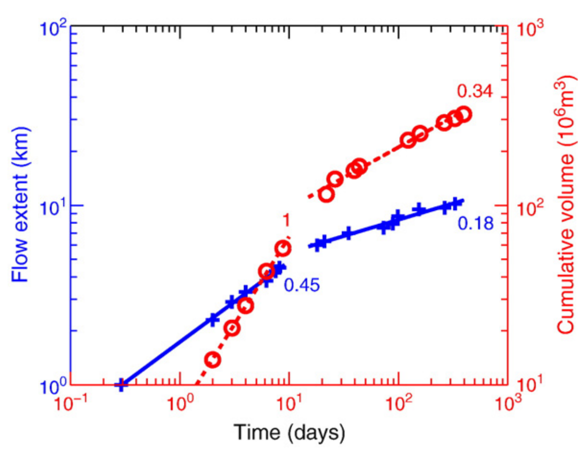

Fig. 4. Extent of the advancing front (+) and the cumulative volume (o) plotted against time on logarithmic scales for the main flow on Lonquimay, 1988–1989. Numbers represent the slope of the corresponding line of best fit. The cumulative volume scaled like t initially and $t^{0.34}$ subsequently, resulting in flow extending a distance proportional to $t^{0.45}$ initially and $t^{0.18}$ subsequently. Data points are from Fig. 3 and table 3 of (Naranjo et al., 1992).

Detailed measurements of a long lava flow on Mount Etna in 2001 can be analysed in a similar fashion. A fissure close to Monte Calcarazzi between 2100 m and 2150 m above sea level, referred to as LFS1 in Coltelli et al. (2007), opened at 2:20 on 18 July 2001. The main lava flow continuously descended from the LFS1 vent and attained its lowest elevation of 1040 m on 25 July 2001, when the lava flow extended a distance of 6.4 km with a maximum width of 545 m. The major channel of lava was supplied predominantly from the LFS1 vent and minimally from a minor flow extending from another vent, LFS2. The cumulative volume of lava in the main LFS1 flow is presented in table 8 of Coltelli et al. (2007), which is approximated by Eq. (3) with $\alpha=1.27$. Likewise, the extending length of the flow is found to scale like $t^{0.50}$. Given that $\alpha=1.27$ and $c=0.50$, the theory developed previously predicts $\beta=2.04$. The apparent viscosity of the lava increased like $t^{2.04}$, suggesting that the flow decelerated considerably due to thermal effects (Fig. 5).

4. Summary

The initial advance of long lava flows was investigated quantitatively using scaling arguments. The extending length of the lava was shown theoretically to depend importantly on the supply from upstream and minimally on the shape of the channel. Thermal effects on the bulk flow were examined to play a major role at subsequent stages when the flow front decelerates considerably. The use of scaling arguments, as drawn out in this paper, can be extended to incorporate additional effects or even investigate new problems, including ones for which it is very difficult to write down the governing equations completely.

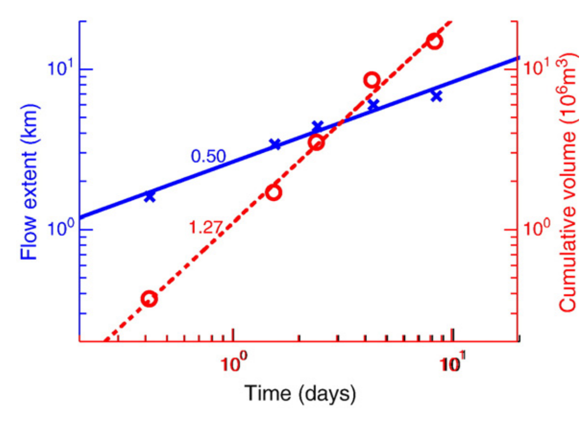

Fig. 5. Extent of the advancing front (x) and the cumulative volume (o) of lava both plotted against time on logarithmic scales during the early stages of the LFS1 flow on Etna in 2001 (Coltelli et al., 2007). Numbers indicate the slope of the corresponding line of best fit.

The theoretical predictions were applied to explain the initial development of natural lava flows on the volcanoes of Etna, Kilauea and Lonquimay. We determined that selected lava flows on Kilauea extended initially like a Newtonian fluid of constant viscosity, while lava flows on Etna and Lonquimay increased considerably in viscosity throughout the course of their propagation. This suggests that thermal effects due to cooling played a major role at all times as the flows advanced on Etna and Lonquimay but only after the flows nearly attained their maximal extent on Kilauea.

Our analysis of field observations demonstrates how the morphology of long lava flows can be studied readily using quick and simple techniques. The relationship $\beta = 1 + 2\alpha - 3c$ allows one of the three parameters (α, β, c) to be calculated given the other two, where the cumulative volume $\sim t^{\alpha}$ and viscosity $\sim t^{\beta}$ of the lava extends a distance $\sim t^{c}$. This relationship enables any variation in the apparent viscosity of the lava, a useful indicator of the role of heat loss, to be estimated solely by monitoring the growth rates of the cumulative volume and the flow extent. Values of α and β characterising active lava flows in the early stages provide an estimate of c, helpful for predicting any further advance of the lava.

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