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Instability of a gravity current within a soap film

Raymond E. Goldstein[1,†], Herbert E. Huppert[1,2,3], H. Keith Moffatt[1] and Adriana I. Pesci[1]

[1]Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

[2]School of Earth Sciences, University of Bristol, Bristol BS8 1TW, UK

[3]School of Mathematics and Statistics, University of New South Wales, Sydney NSW 2052, Australia

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One of the simplest geometries in which to study fluid flow between two soap films connected by a Plateau border is provided by a catenoid with a secondary film at its narrowest point. Dynamic variations in the spacing between the two rings supporting the catenoid lead to fluid flow between the primary and secondary films. When the rings are moved apart, while keeping their spacing within the overall stability regime of the films, after a rapid thickening of the secondary film the excess fluid in it starts to drain into the sloped primary film through the Plateau border at which they meet. This influx of fluid is accommodated by a local thickening of the primary film. Experiments described here show that after this drainage begins the leading edge of the gravity current becomes linearly unstable to a finite-wavelength fingering instability. A theoretical model based on lubrication theory is used to explain the mechanism of this instability. The predicted characteristic wavelength of the instability is shown to be in good agreement with experimental results. Since the gravity current advances into a film of finite, albeit microscopic, thickness this situation is one in which the regularization often invoked to address singularities at the nose of a thin film is physically justified.

Key words: fingering instability, interfacial flows (free surface), lubrication theory

1. Introduction

One of the iconic problems in singularity formation is the collapse of a soap film catenoid when the two rings supporting it are pulled apart (Plateau 1873; Cryer &

† Email address for correspondence: R.E.Goldstein@damtp.cam.ac.uk

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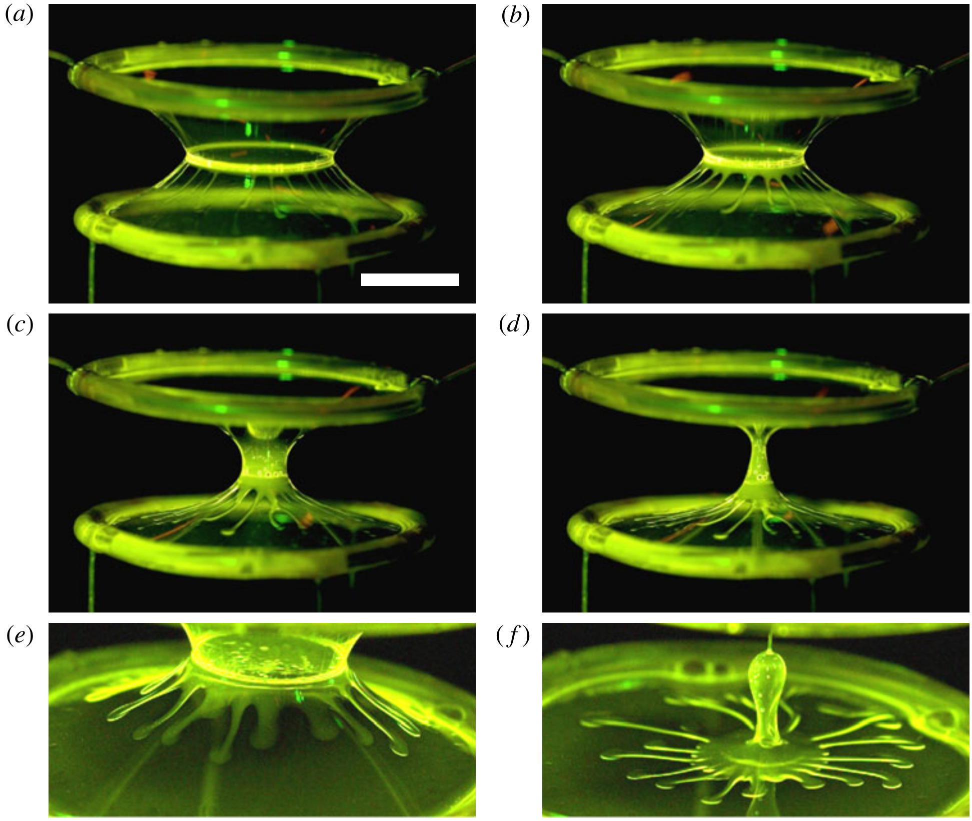

FIGURE 1. Snapshots of a collapsing catenoid with a secondary film. A fingering instability occurs as fluid flows out of the central disc, through the Plateau border, into the lower section of the catenoid. Images taken at 2500 frames s^{-1}. Defining time $t=0$ in (*a*), images are at (*b*) $t=223$ ms, (*c*) 253 ms and (*d*) 257 ms. Scale bar is 1 cm. Panel (*e*) shows, for a different collapse event than (*a–d*), fully developed plumes exhibiting clear 'mushroom caps'. In panel (*f*), after the collapse, the central disc has become part of the film attached to the lower ring; its remnant is the fluid-rich region in the centre which constituted the reservoir from which the plumes emanated.

Steen 1992; Chen & Steen 1997; Robinson & Steen 2001). The complex dynamics of the pinching singularities that lead to topological rearrangements are primarily a consequence of the film's surface tension and air pressure (Keller & Miksis 1983), with little dependence on the inner structure of film. It is well known that the same frame can also support a catenoid with a second soap film at its waist (Almgren 1966). This arrangement has recently been used to study the dynamics by which new film is generated when the frame spacing is changed by small amounts (Seiwert *et al.* 2013), where the catenoid and the central film are always stable and the rings holding the film are drawn closer together. This produces growth in the radius of the central disc, making it possible to observe the development of new film from the Plateau border. These results lead naturally to the question of how separating the rings, instead of drawing them together, affects the behaviour of the film near the Plateau border of the central disc and also of how the collapse of the catenoid is changed, if at all, by the presence of the secondary film. Figure 1 shows snapshots from a high-speed movie of experiments we performed (see § 2) to address these questions. It shows a full collapse event (see also supplementary movie 1 available at http://dx.doi.org/10.1017/jfm.2014.395) when the catenoid has an additional circular film located midway between the rings, where the film has been prepared and illuminated to enhance contrast (see § 2). As the collapse occurs, the inner circular film shrinks, losing part of its fluid to the retracting Plateau border. After the onset of the instability that leads to collapse, fingers develop just below the disc's Plateau border, and evolve to become plumes that extend from the Plateau border downwards within the bottom half of the catenoid. Notice that the advancing tip of each plume tends to form a 'mushroom cap' structure. In the final stages of the collapse the remainder of the central disc turns into a long cylindrical bridge that breaks apart, producing a satellite drop (Leppinen & Lister 2003), unlike in the case of a simple catenoid, where the satellite 'drop' is in reality a bubble. It is important to note that the fingering instability occurs independently of the collapse. As shown in figure 2 (see also supplementary movie 2) it is possible to induce the incipient formation of fingers even in the case when the film moves in a controlled manner.

Motivated by the observations shown in figures 1 and 2, we discuss different theoretical scenarios for the origin of these patterns. In § 3 we first analyse the stability of the retracting border itself, approximating the central disc as a Hele-Shaw cell whose thickness increases as a function of time (Shelley, Tian & Wlodarski 1997). The border is found to be marginally stable when contracting under the action of surface tension at its edge, ruling out an instability of the border itself as the cause of the fingers. A second approach considers the inflow into the lower section of the catenoid, due to the retraction of the Plateau border, within the lubrication approximation. The resulting equations describe a gravity-driven current down an incline (Huppert 1982; Troian *et al.* 1989; Duffy & Moffatt 1995; Bertozzi & Brenner 1997; Kondic 2003; Takagi & Huppert 2010), which becomes unstable with a characteristic wavelength whose value is in good agreement with that observed experimentally.

2. Experimental methods

The soap films were composed of a mixture of water, glycerol and Fairy washing-up liquid with a substantial amount of fluorescein added to enable detailed imaging at high speed. The soap film viscosity was $\mu \sim 4\mu_w$, where μ_w is the viscosity of water. Multiple high-power LEDs (Luxeon Star, cyan), each mounted on a finned heat sink, were shone on the film. Images were captured with a Phantom V641 high-speed colour video camera at a resolution of 1920 pixels $\times$ 1080 pixels, equipped with a Zeiss 60 mm f/2 macro lens fitted with a bandpass filter centred on the emission peak of fluorescein. Soap films were supported on circular wire frames covered with 3 mm diameter Tygon tubing. The lower ring is held stationary by three fine wire supports from below, while, as in previous experiments on soap films with unusual topologies (Goldstein *et al.* 2010, 2014), precise manipulation of the upper ring was done by hand. In figure 1, for example, the rings were brought just beyond the point of instability and held in place for the duration of the collapse ($\sim$0.3 s). In figure 2, the upper ring was slowly moved up at a speed (determined directly from the high-speed video) of 2.1 cm s^{-1} and then held in place while the instability developed.

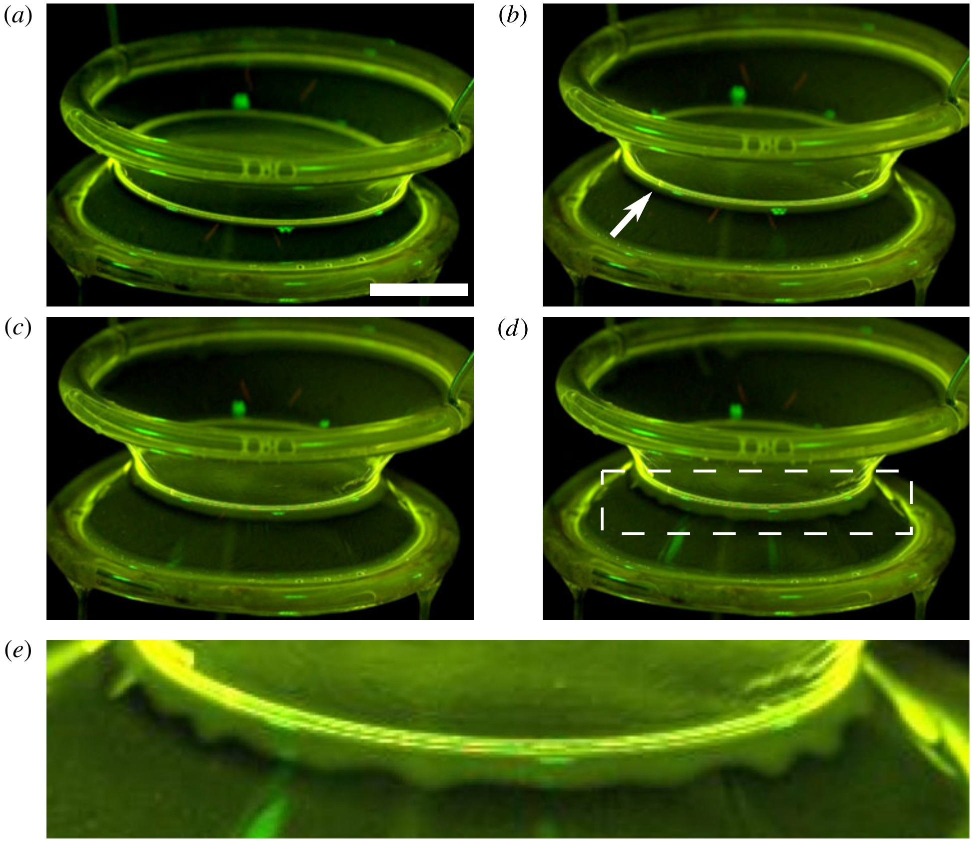

FIGURE 2. Initiation of fingering instability. The panels show the development of a wavy leading edge to the descending front within the soap film as the two supporting rings are moved apart, imaged at 1000 frames s^{-1}. (*a*) Film after a period of equilibration (approximately 30 s after removing the frame from the bath). We define the frame in panel (*a*) as the origin of time t. (*b*) $t = 56$ ms, (*c*) $t = 119$ ms, (*d*) $t = 181$ ms. The arrow in (*b*) indicates black film. Scale bar is 1 cm. Panel (*e*) shows a blow-up of the region inside the dashed box in (*d*), where the onset of the instability is clear.

3. Theory

In this section we consider two possible explanations for the fingering instability. The first is that of an instability of the receding Plateau border itself, while the second is an instability of the front of fluid advancing down the lower section of the catenoid.

3.1. *Marginal stability of a contracting disk*

As the supporting rings are separated and the waist of the catenoid contracts, the dynamics of the secondary film consists primarily of a shrinkage in radius, compensated by a growth in thickness, its volume being approximately constant. As analysed by Aradian, Raphaël & de Gennes (2001) in their studies of 'marginal pinching', when the water–air interfaces of a soap film are heavily loaded with surfactants and the Marangoni stresses are large, the surfaces obey a no-slip boundary condition. (The alternative scenario has been investigated by Howell & Stone, 2005.)

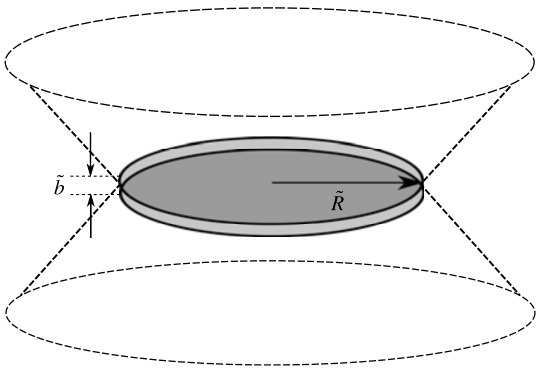

FIGURE 3. Geometry of the contracting secondary film.

The contraction problem is then very similar to the motion of a drop of fluid confined between two plates in the classic Hele-Shaw configuration, where the gap between the plates is increased as a function of time. The contraction of such a drop has been studied for an arbitrary time-dependent gap width $b(t)$, where t is the time, and the conditions under which the drop has a fingering instability have been established (Shelley *et al.* 1997). This fingering instability is triggered by the penetration of air into a region of viscous fluid in the classic Saffman–Taylor context (Saffman & Taylor 1958). One might imagine that such a fingering instability could be the cause of the plumes and undulations depicted in figures 1 and 2. In the earlier work by Shelley *et al.* (1997) the time-dependent gap was an arbitrary function of time. Here we note that once the entire film (catenoid plus central disc) is no longer in equilibrium, its motion will be dominated by the surface tension force associated with the largest curvature, which is that of the edge of the disc. We use that Laplace pressure to determine the rate of shrinkage of the drop radius. Intriguingly, we find that the resulting $b(t)$ is a power law with an exponent that renders the shrinking disc marginally stable.

Using the geometry of figure 3, we consider the depth-averaged lubrication approximation of the fluid velocity $\tilde{\boldsymbol{u}}$ within the disc of radius $\tilde{R}$ and thickness $\tilde{b}$,

$$\tilde{\boldsymbol{u}} = -\frac{\tilde{b}^2}{12\mu}\boldsymbol{\nabla}\tilde{p}, \tag{3.1}$$

where $\boldsymbol{\nabla}$ is the two-dimensional gradient in the plane of the disc, μ the dynamic viscosity and $\tilde{p}$ the pressure. This is supplemented by the divergence condition on the fluid velocity that arises from incompressibility and the changing thickness,

$$\boldsymbol{\nabla}\cdot\tilde{\boldsymbol{u}} = -\dot{\tilde{b}}/\tilde{b}, \tag{3.2}$$

where $\dot{} = \mathrm{d}/\mathrm{d}t$. Finally, the boundary condition at the drop edge is that of the Laplace pressure,

$$\tilde{p}(\tilde{R}) = \tilde{\sigma}/\tilde{R}, \tag{3.3}$$

where $\tilde{\sigma}$ is the surface tension. In arriving at this form, we have ignored the second contribution to the mean curvature at the disc edge. This is possible because before the collapse begins the inward force due to the curvature $1/R$ is balanced by the resultant of the tensions in the two catenoids (which themselves can be viewed as the depth-averaged contribution from the curvature of the disc edge). Once the film is rendered unstable, the $1/R$ curvature (which we have focused on) must dominate (and more so as R decreases) as this is the cause of the collapse instability.

If we use the initial radius $\tilde{R}_0$ as a characteristic length to scale the in-plane coordinates, the initial thickness $\tilde{b}_0$ to scale $\tilde{b}$, and the ratio $t_0 = \tilde{b}_0/|\dot{\tilde{b}}_0|$ to define a characteristic time, then the rescaled equations are

$$\boldsymbol{u} = -b^2\nabla p, \tag{3.4a}$$
$$\nabla\cdot\boldsymbol{u} = -\dot{b}/b, \tag{3.4b}$$

where $\dot{}$ now denotes $\mathrm{d}/\mathrm{d}\tau$, with $\tau = t/t_0$, and $p(R) = \sigma/R$, where $\sigma = \tilde{\sigma}\tilde{b}_0^3/(12\mu|\dot{\tilde{b}}_0|\tilde{R}_0^3)$.

Substituting (3.4a) into (3.4b), we obtain

$$\nabla^2 p = \dot{b}/b^3, \tag{3.5}$$

with particular solution $p(r) = \dot{b}r^2/(4b^3)$. Once the catenoid is unstable, the motion of the film and shrinkage of the disc are driven by the curvature of the neck of the catenoid. Hence, invoking the boundary condition and using the conservation law $R^2 b = 1$ yields the evolution equation for $R(\tau)$

$$R^6\dot{R} = -2\sigma, \tag{3.6a}$$
$$\text{with solution } R^7(\tau) = 1 - 14\sigma\tau. \tag{3.6b}$$

This vanishing of the radius is accompanied by a power-law growth of the thickness, $b = (1 - 14\sigma\tau)^{-2/7}$. In the original variables the radius evolution takes the form

$$\tilde{R}(t) = \tilde{R}_0\left(1 - \frac{t}{\tau_c}\right)^{1/7}, \tag{3.7}$$

where $\tau_c = 6\mu\tilde{R}_0^3/(7\tilde{\sigma}\tilde{b}_0^2)$.

The stability analysis (Shelley *et al.* 1997) assumes perturbations of the form $\exp(\mathrm{i}qs)$, where $s = R\theta$ is the arclength around the unperturbed circle and θ is the polar angle. The growth rate β of a perturbation of dimensionless wavenumber $k = qR$ is

$$\beta(k,\tau) = \frac{k}{2}\frac{\dot{b}}{b} + \frac{b^2}{R^3}\sigma\left(k - k^3\right), \tag{3.8}$$

where β now depends on the rescaled time τ through the time-dependent b and R. The mode $k = 1$ corresponds to an infinitesimal translation, and $\beta = 0$ identically for $k = 1$, as required by translational invariance. This implies a band of instability

$$0 < k^2 < \frac{\dot{b}R^3}{2b^3\sigma}. \tag{3.9}$$

With the equations of motion above, we find that $\dot{b}R^3/2b^3\sigma = 2$, so that not only is the width of the unstable band of modes independent of time, but, since k must be an integer greater than unity, there is in fact no unstable mode. Because the jump in slope of the catenoid at the secondary film will slow down the contraction, the real system should be more stable than the simplified ideal model. These facts strongly suggest that there is no azimuthal instability of the secondary film when it contracts under its own surface tension.

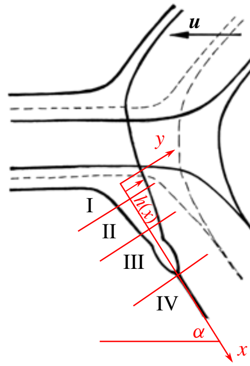

FIGURE 4. Cross-sectional geometry of films in the connection region between the central disc and the catenoid during retraction of the disc.

3.2. *Lubrication theory for film flow*

The typical structure of a film that is drawn from a reservoir was studied and described in detail by Mysels, Shinoda & Frankel (1959). They found that after a short period of stabilization (30–120 s) the film develops a 'hollow ground' or black film and that the cross-section of the drawn film nearest the Plateau border displays four different regions. As shown in figure 4, the first of these regions (I) corresponds to the section of the border retracting with velocity $U(t)$ and has an approximately constant curvature. This region is followed by a transition zone (II) of high viscous dissipation with large volume flow, but with a slowly varying slope, which connects to a zone (III) where the slope of the film is still slowly varying and the flows within have no interaction with the flows from preceding regions. Finally, there is a stagnant thin film (IV) of negligible thickness (Schwartz & Princen 1987; Schwartz & Michaelides 1988, 1989), as indicated by the white arrow in figure 2(*b*). The characteristics of this type of film make it possible to apply the lubrication approximation to find an equation of motion to determine the profile of the film in the regions with a slowly varying slope.

In our case, the catenoid with the disc has a profile with a black film section, as seen in the experimental figure 2 and shown schematically in figure 4. The geometry of the attachment region between the secondary film and the catenoid is of the same type as that studied by Mysels *et al.* (1959, Chapter IV), except that the lower film has a finite slope. Once the thin black film has formed and the central disc starts contracting, there will be a low flow into the Plateau border. As the contraction continues, the thickness of the disc will increase to compensate for the loss of area, and the radius of curvature of the Plateau border will start to increase. When the driving pressure is large enough the outer interface in region I (figure 4) will bulge outwards and its curvature will change sign, with the consequence that liquid will spill along the catenoid's section. This overflow can be understood quantitatively to be a consequence of the pressure difference generated near the Plateau border when a film is drawn, as discussed by Mysels *et al.* (1959) and others (Landau & Levich 1942; Bretherton 1961). The critical height for this curvature change is $h_0 = (2\sigma/\rho g)^{1/2}$. The value of h_0, which for a soap solution is approximately 1/4 cm, determines the maximum possible thickness of the secondary film (central disc). As the thickness of the disc exceeds h_0 there is an excess pressure pushing the fluid into the Plateau border. Any further shrinkage of the radius of the central film will produce a spill over of fluid through the Plateau border and along the slope of the lower section of the catenoid. Since, as noted in the previous subsection, the central film does not have an unstable edge, the fingers that develop in the lower section of the catenoid are a consequence of an instability of this gravity current. It follows that the finite speed of contraction of the disk serves only to accentuate and accelerate the development of the instability, rather than to be its cause.

To study the fingering process we apply lubrication theory in a similar manner to the original work of Huppert (1982) for the case of thin viscous flows down inclines (see also Barenblatt, Beretta & Bertsch, 1997). In the present situation the gravitational effects in the direction perpendicular to the length of the film can be neglected and, thus, the profile of the film will be considered to be symmetric $y \to -y$ in the coordinate system shown in figure 4, where the half-thickness of the film is $h(x)$, which is inclined at an angle α with respect to the horizontal. As a first approximation we assume the soap film to be rigid, implying a no-slip condition (Aradian *et al.* 2001). This assumption is supported by our experiments, in which tracer particles on the surface remain mostly stationary during the entire process. Applying this boundary condition at $y = h$ and the symmetry condition $u(h) = u(-h)$ to calculate the flux Q within the lubrication approximation, the evolution equation takes the form $h_t = -Q_x$,

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left(\frac{2\sigma}{3\mu} h^3 \frac{\partial^3 h}{\partial x^3} + \frac{2\rho g \sin\alpha}{3\mu} h^3\right). \tag{3.10}$$

If we choose the point $x = 0$ to be at the start of region II, we impose there the additional boundary condition

$$Q|_{x=0} = 2Uh(0, t) \tag{3.11}$$

to take into account the influx of fluid coming from the Plateau border overflow. Such forced lubrication dynamics have been considered previously (Schwartz & Michaelides 1988).

Rescaling the coordinates and the flux as $x = L_x \bar{x}$, $h = L_y \bar{h}$ and $q = Q/(UL_y)$, we write the expression for the flux within the layer as

$$q = 2\bar{h} + \frac{2\sigma}{3\mu U}\left(\frac{L_y}{L_x}\right)^3 \bar{h}^3 \frac{\partial^3 \bar{h}}{\partial \bar{x}^3} + \frac{2\rho g \sin\alpha}{3\mu U} L_y^2 \bar{h}^3. \tag{3.12}$$

The balance between all three terms leads to the following characteristic lengths:

$$L_x \sim \left(\frac{2\sigma}{3\mu}\right)^{1/3}\left(\frac{3\mu}{2\rho g \sin\alpha}\right)^{1/2} U^{1/6} \quad \text{and} \quad L_y \sim \left(\frac{3\mu}{2\rho g \sin\alpha}\right)^{1/2} U^{1/2} \tag{3.13a,b}$$

where the typical value of the velocity of influx will be approximately the same as the velocity given by the rate of contraction of the disc calculated in the previous section, $U \sim |u_r(R)| = |\dot{R}|$. Thus, we find $u_r = -U_0(1 - t/\tau_c)^{-6/7}$, where $U_0 = (\sigma/6\mu)(b_0/R_0)^2$. This yields a relationship between the thickness of the film and the parameters of the central disc,

$$L_y \simeq \left(\frac{\sigma}{4\rho g \sin\alpha}\right)^{1/2} \frac{b_0}{R_0}, \tag{3.14}$$

where L_y is half the thickness of the layer.

In our experimental set-up the two rings that hold the film have been separated so that the central disc shrinks without any further vertical motion of the rings. Thus, the bottom section of the film moves very little in the vertical direction, so that the central disc can be considered to be vertically stationary. Under these conditions the critical value for b is well approximated by $b \sim h_0 \sim 2.5$ mm. The critical velocity which will balance the pressure gradient due to the surface tension is $U \sim 0.2$ cm s^{-1}. Given that the retraction of the central disc produces a flow that has a velocity which we measure to be $\mathrm{d}R/\mathrm{d}t \sim 3$ cm s^{-1} (with downslope flows reaching $\sim$10 cm s^{-1}), the gravity current is unstable from the very beginning of the contraction and as soon as the height of the disc has reached the maximum value allowed by the balance of pressures. Using the same argument advanced by Huppert (1982) and also supported by other analyses (Troian *et al.* 1989; Jerrett & de Bruyn 1992) we find the estimated value of the wavelength under these conditions is $\lambda = 2\pi L_y$, which yields $\lambda \simeq 0.35$ cm, in good agreement with the experimental value $\lambda \simeq 0.32$ cm. Because the perimeter of the central disc is $2\pi R_0 \gg \lambda$ the requirement that there should be an integer number of wavelengths in the perimeter is within the error of the approximation. The measured growth rate of the instability is $\beta \simeq 14$ s^{-1}. This agrees well with the prediction of Troian *et al.* (1989), $\beta \simeq 0.5 U_0/\ell \sim 15$ s^{-1}, where we have used the experimentally determined values $U_0 \sim 3$ cm s^{-1}, and, for the typical distance the front has moved before fingering, $\ell \simeq 0.1$ cm.

4. Conclusions

In this work we report the novel finding of a gravity current in a soap film becoming unstable to a fingering instability. With exception of the boundary conditions and the forcing that drives the system, the underlying physics and mathematics of this current are similar to those in other, more familiar cases (Huppert 1982; Troian *et al.* 1989; Duffy & Moffatt 1995; Bertozzi & Brenner 1997; Kondic 2003; Takagi & Huppert 2010). The existence of this instability illustrates one more aspect of the complex issues that are connected with flows through Plateau borders (Seiwert *et al.* 2013) and within soap films themselves. The study of the two possible sources of the instability makes it clear that the fingering is not a product of the increase in thickness of the central disc, as in the case of time-dependent Hele-Shaw flow, but a consequence of the liquid overflow and its subsequent 'fall' down the sloped bottom half of the catenoid. It is also noteworthy that the simplest theoretical approach yields such a good agreement with the experimental results for the characteristic wavelength of the fingering.

An interesting issue left unexplored is the study of the evolution of the plumes that appear after the instability has developed fully: in particular the 'mushroom' caps that constitute the advancing front of the plume (figure 1). Unlike the case of a gravity current flowing down an incline, where the front has discontinuous boundary conditions which are often artificially smoothed out via a precursor film to make it possible to find a solution (de Gennes 1985), the boundary conditions for the plumes' fronts within the soap film are naturally smooth, as they connect to the thin black film ahead, making it a much easier problem to solve in spite of its three-dimensional character (Barenblatt *et al.* 1997).

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Supplementary movies

Supplementary movies are available at http://dx.doi.org/10.1017/jfm.2014.395.

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