$$E = (2\beta)^{-3/2}\eta^{3/2}k^{-3}.$$

This is the two-dimensional enstrophy cascading equivalent of the three-dimensional energy cascading Kolmogoroff–Obukhov spectrum and can be deduced by applying the usual dimensional argument to enstrophy instead of to energy.

There is some evidence for the existence of -3 power spectra for planetary scales of atmospheric motions[5] and for larger scales of clear air turbulence.[6] In both cases this could be due to two dimensionality of the eddies. However, it should be remembered that a -3 power-law results from dimensional arguments whenever one assumes as the single physically significant constant one having dimensions involving (as does η) time only, and other candidates might be mean wind shear dV/dz or a stability parameter

$$\frac{g}{T}\frac{dT}{dz}.$$

The diffusion approximation is not consistent with the two-dimensional inviscid equilibrium distributions[4]

$$E(k) = 2E_0k_0k/(k_0^2 + k^2),$$

(E_0, k_0 constant) analogous to the Lee distribution for three dimensions.

This work was performed under the auspices of the United States Atomic Energy Commission.

[1] C. E. Leith, Phys. Fluids **10**, 1409 (1967).
[2] T. D. Lee, Quart. Appl. Math. **10**, 69 (1952).
[3] E. Hopf, J. Ratl. Mech. Anal. **1**, 87 (1952).
[4] R. H. Kraichnan, Phys. Fluids **10**, 1417 (1967).
[5] A. Wiin-Nielsen, Tellus **19**, 540 (1967).
[6] N. Z. Pinus, E. R. Reiter, G. N. Shur, and N. K. Vinnichenko, Tellus **19**, 206 (1967).

Numerical Evaluation of the Tietjens Function

Herbert E. Huppert
Institute of Geophysics and Planetary Physics and Department of Aerospace and Mechanical Engineering Sciences, University of California, La Jolla, California
(Received 23 August 1967; final manuscript received 24 November 1967)

The Tietjens function is expressed in terms of two power series, which are much simpler and take significantly less time to calculate than those recently proposed by Chen, Joseph, and Sparrow.

The Tietjens function $F(z)$ is used in the determination of the asymptotic solution of the Orr–Sommerfeld equation (see e.g., Lin[1] and Reid[2]). The most extensive, and most accurate, tabulation at present is given by Miles,[3] who quotes the real and imaginary parts of $F(z)$ for $z = -6(0.1)10$; $4S$. A method of evaluating the Tietjens function as the ratio of two rapidly convergent power series has recently been given by Chen *et al.*[4] The purpose of this note is to present two power series which are simpler, and take less time to compute, than those given by Chen *et al.*[4]

The Tietjens functions may be defined by[5]

$$F(z) = 1 - \mathrm{Ai}'(\zeta)\left[\zeta\int_\infty^\zeta \mathrm{Ai}(\tau)\,d\tau\right]^{-1}, \qquad \zeta = ze^{-5i\pi/6}. \tag{1a,b}$$

We determine the power-series representation of the Airy function (valid in every finite region of the complex plane) by direct substitution into the differential equation, obtaining

$$\mathrm{Ai}(\zeta) = \sum_{n=0}^{\infty} a_n\zeta^n, \tag{2}$$

where

$$a_0 = \mathrm{Ai}(0) = 0.3550\cdots, \quad a_1 = \mathrm{Ai}'(0) = -0.2588\cdots, \quad a_2 = 0, \tag{3}$$

$$a_{n+3} = a_n[(n+2)(n+3)]^{-1}, \qquad n \ge 0. \tag{4}$$

Appropriately differentiating and integrating Eq. (2), and then substituting the results into Eq. (1a), we obtain

$$F(z) = 1 - \left(\sum_{n=0}^{\infty} b_n\zeta^n\right)\left(-\tfrac{1}{3}\zeta + \sum_{n=0}^{\infty} c_n\zeta^{n+2}\right)^{-1}, \tag{5}$$

where

$$b_0 = \mathrm{Ai}'(0), \qquad b_1 = 0, \qquad b_2 = \tfrac{1}{2}\mathrm{Ai}(0), \tag{6}$$

$$b_{n+3} = b_n[(n+1)(n+3)]^{-1}, \qquad n \ge 0, \tag{7}$$

$$c_0 = \mathrm{Ai}(0), \qquad c_1 = \tfrac{1}{2}\mathrm{Ai}'(0), \qquad c_2 = 0, \tag{8}$$

$$c_{n+3} = (n+1)c_n[(n+2)(n+3)(n+4)]^{-1}, \qquad n \ge 0. \tag{9}$$

We find that the representation of the Tietjens function as given by (5) is much simpler than that of Chen *et al.*[4] and can be calculated in approximately two-thirds the time. It is in a form ideally suited for inclusion as a subroutine in a digital computer program whose purpose is to determine the asymptotic solution of the Orr–Sommerfeld equation.

This work was performed while the author was the holder of a Sydney University Post-Graduate Travelling Fellowship; the computing was financed by National Science Foundation Grant GA-849.

[1] C. C. Lin, in *The Theory of Hydrodynamic Stability* (Cambridge University Press, Cambridge, England, 1966), Chaps. 3, 5, and 8.

[2] W. H. Reid, in *Basic Developments in Fluid Mechanics*, M. Holt, Ed. (Academic Press Inc., New York, 1965), Chap. 3.
[3] J. W. Miles, J. Fluid Mech. **8**, 593 (1960).
[4] T. S. Chen, D. D. Joseph, and E. M. Sparrow, Phys. Fluids **9**, 2519 (1966).
[5] The numerator of Eq. (3.72), Ref. 2, p. 273, has been integrated once by parts.

Development of Magnetohydrodynamic Waves behind an Ionizing Shock Wave

M. D. Cowley* and S. J. Morin†
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts
(Received 20 June 1967; final manuscript received 6 December 1967)

A transverse perturbation to a longitudinal magnetic field is introduced upstream of an ionizing shock. The dominant length scale is given by Ohmic diffusion. Either a transverse structure develops, and one wave is emitted, or two slow waves are emitted.

In a previous paper[1] a model was suggested for the structure of ionizing shock waves when there is a magnetic field present and the length scale for Ohmic diffusion is much larger than the length scales for viscous diffusion and relaxation to equilibrium ionization. An initial transition, which raises the temperature, ionizes the gas, and changes the electrical conductivity from zero to a finite value, has the character of an ordinary gasdynamic shock, being too thin for magnetic forces to affect the transverse momentum or for currents to affect the field (see Fig. 1). Subsequently, magnetic forces and currents may have an influence in a broad region (QR), which can be treated as inviscid. It was shown that the region QR can only exist if the downstream normal velocity u is less than the slow magnetohydrodynamic wave speed c_s given by the smaller positive root of

$$c^2 = \tfrac{1}{2}\{(a^2 + b^2) \pm [(a^2 + b^2)^2 - 4a^2b^2 \cos^2 \theta]^{\frac{1}{2}}\}, \tag{1}$$

where a is the sound speed in the absence of a magnetic field, b is the Alfvén-wave speed equal to $B/(\mu\rho)^{\frac{1}{2}}$, and θ is the angle between the shock normal and direction of magnetic field. If QR exists, the downstream state cannot be uniquely determined from the upstream velocity, magnetic field, and gas state, while it can be so determined if QR does not exist. To reinforce the previous theory, we consider the detailed evolution of the structure of an ionizing shock wave in a particular and simple situation.

The equations for tranverse motion and Ohm's law for one-dimensional, time-dependent situations

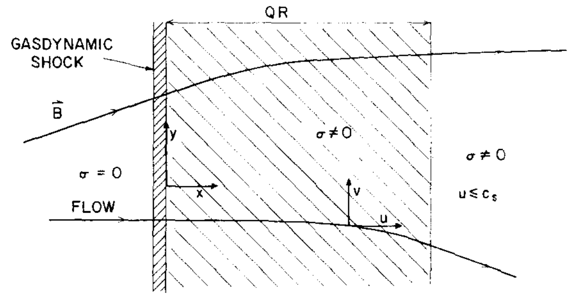

Fig. 1. Ionizing shock with broad Ohmic-diffusion region QR. The choice of frame of reference gives a stationary shock wave.

in a finitely conducting gas give

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} - \frac{B_x}{\mu_0} \frac{\partial B_y}{\partial x} = 0, \tag{2}$$

$$\frac{\partial B_y}{\partial t} + u \frac{\partial B_y}{\partial x} - B_x \frac{\partial v}{\partial x} = \lambda \frac{\partial^2 B_y}{\partial x^2}, \tag{3}$$

where λ is the magnetic diffusivity equal to $1/\mu\sigma$ and the remaining symbols have their usual meaning, coordinates and components being indicated in Fig. 1. Suppose that, for $t < 0$, $v = B_y = 0$ everywhere so that the shock only consists of the gasdynamic transition (no region QR). Then a small, constant, tranverse field is switched on in the nonconducting gas, giving initial conditions for the problem

$$v = 0 \quad \text{for} \quad -\infty < x < \infty; \qquad B_y = B_{y0} \quad \text{for} \quad x \le 0; \qquad B_y = 0 \quad \text{for} \quad x > 0 \quad \text{at} \quad t = 0. \tag{4}$$

It must be imagined that the return path for the current sheet formed at $x = 0$, $t = 0$ is closed at $x \to -\infty$. We also assume that subsequently B_y is small enough for changes in magnetic pressure to have negligible effect in the longitudinal equation of motion, and changes in v cause negligible variation in kinetic energy. Thus the longitudinal motion is decoupled; ρ, u, and λ are not affected by the transverse perturbation, and we assume that they are constant. To follow the subsequent development of the current sheet, Eqs. (2) and (3) can be solved for the region $x > 0$ subject to initial conditions (4) and boundary conditions

$$v = 0, \quad B_y = B_{y0} \quad \text{at} \quad x = 0; \qquad v, B_y \to 0 \quad \text{as} \quad x \to \infty. \tag{5}$$

We have used the same method of solution as Todd,[2] who analyzed perturbations to shock waves with conducting gas on both sides. The Laplace transform of B_y was found and inverted numerically. Results for conditions such that $b = 2u$ downstream are shown in Fig. 2 and for $b = \frac{1}{2}u$ in Fig. 3. The