The Melting of Ice in Cold Stratified Water

HERBERT E. HUPPERT

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW England

EDWARD G. JOSBERGER

School of Oceanography, Oregon State University, Corvallis, 97331

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ABSTRACT

We consider the melting of ice in cold water vertically stratified with salt. The study extends previous investigations of ice melting in cold water at uniform salinity and in warm water with a salinity gradient. We find, in agreement with the results of the latter study, that the meltwater spreads out in a series of horizontal layers. This motion tends to convert the initially smooth salinity distribution into one with much larger gradients in the interfaces between the layers. The thickness of the layers is well represented by

$$h = 0.65[\rho(T_{fp},S_\infty) - \rho(T_\infty,S_\infty)]\left(\frac{d\rho}{dz}\right)^{-1},$$

where T_{fp} is the freezing point at the mean far-field salinity and $d\rho/dz$ is the vertical density gradient due to salinity. We also discuss the results of an experiment with a stratified region above a region of uniform salinity. The convective plume that formed in the lower region penetrated into the upper region, while a series of horizontal layers in the upper region extended beyond the plume. We conclude the paper with a discussion of the application of our experimental results to oceanographic conditions.

1. Introduction

The formation of ice by freezing during the winter in polar seas, and the associated exclusion of salt, is well known to be fundamental to the physical oceanography of the Arctic and Antarctic Oceans. However, the contrasting influence of the melting of icebergs has been the subject of comparatively few investigations. Icebergs form when glaciers flow out over land into the adjoining sea and break off, or calve. Each year approximately 5000 icebergs are calved into the Antarctic Ocean in this way. These icebergs initially have horizontal sizes of typically 1 km and their initial depth is typically 250 m, the mean thickness of the glacier or ice shelf at the point of calving. Each iceberg thus contains of the order of 10^8 m^3 or 10^{11} kg of frozen, remarkably fresh, water. Arctic icebergs, calved mainly from the glaciers on the west and east coast of Greenland and from the glaciers of Ellesmere Island, are much more numerous, though smaller. Approximately 25 000 are calved each year with a typical depth of 50 m. Those calved on the east coast of Greenland drift southward in the prevailing current, round Kap Farvel and then drift northward to be joined by the icebergs calved from the west coast glaciers. This stream of icebergs floats westward across Baffin Bay, is joined by those originating from the southern coast of Ellesmere Island and then migrates southward in the Labrador Current and on toward the Grand Banks of Newfoundland. Only about 400 icebergs, however, manage to complete the journey and appear at the Grand Banks and occasionally in the main North Atlantic shipping lanes. It has been argued that the melting of these icebergs takes place preferentially along the sides, rather than at the top or bottom (Huppert and Turner, 1980), and it is the process of the melting of a vertical ice wall on which we shall concentrate in the present paper.

The work reported is a continuation of two previous laboratory investigations, accompanied by theoretical considerations, of such melting. In one the ice melted in water of uniform salinity (Josberger, 1979; Josberger and Martin, 1980) and in the other a vertical salinity gradient stratified the surrounding water, which was at a temperature of around 20°C (Huppert and Turner 1978, 1980). In the remainder of this section we describe the principal results of these two studies and the main questions they left unanswered. In the next section we present the results of our current experiments. This is

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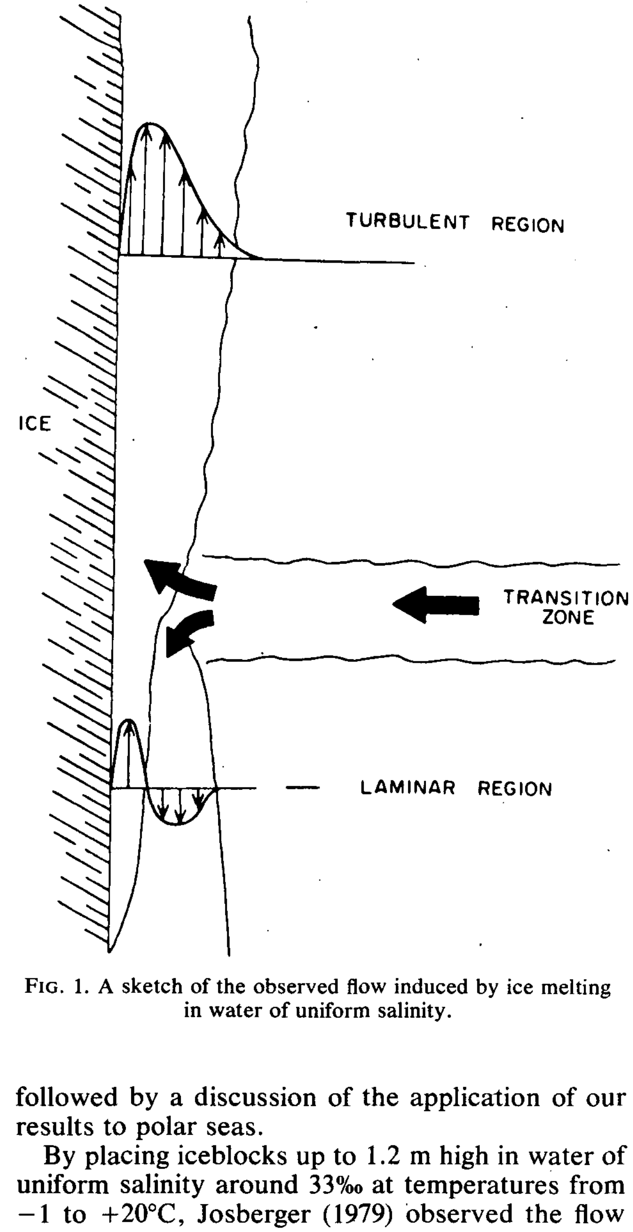

FIG. 1. A sketch of the observed flow induced by ice melting in water of uniform salinity.

followed by a discussion of the application of our results to polar seas.

By placing iceblocks up to 1.2 m high in water of uniform salinity around 33‰ at temperatures from −1 to +20°C, Josberger (1979) observed the flow pattern sketched in Fig. 1. Near the bottom of the ice wall, where the flow was laminar, there was an upward flowing inner layer and a downward flowing outer layer. The inner layer contained dilute cold water which was lighter than the ambient salty water, while the outer layer contained salty cooled water which was heavier. This buoyancy distribution resulted from the large difference between the diffusivities of heat and salt. The pattern persisted until the inner layer became turbulent. Turbulence occurred when the overall Grashof number

$$G_0 = g[\rho(T_\infty,S_\infty) - \rho(T_w,S_w)]l^3\nu^{-2}, \qquad (1)$$

based on the length l of the boundary layer, achieved a value of about 2×10^8; here $\rho(T,S)$ is the density as a function of temperature and salinity and the subscripts w and ∞ indicate evaluation at the wall and in the farfield, respectively. For oceanographic values of the parameters, l is less than 0.5 m and the purely laminar region would be confined to a very minor fraction of the ice wall.

Above the region of purely laminar flow, there was a turbulent boundary layer having upward flow only. The fluid in this layer was lighter than the ambient fluid because the turbulence increased the diffusion of dilute water away from the ice, where the dilution effects overwhelmed cooling effects. Thermistors frozen into the ice recorded a vertically uniform T_w adjacent to the turbulent region, which, at a salinity of 33‰, was empirically related to T_{fp}, the freezing point based on the farfield salinity (~ −1.9°C at 33‰), by

$$T_w = T_{fp}\{1 - \tanh[\alpha(T_\infty - T_{fp})]\}, \qquad (2)$$

where $\alpha = 0.15$ (°C)$^{-1}$. Thus when T_∞ was greater than a value of around 10°C, $T_w \approx 0$, and as T_∞ approached T_{fp} so did T_w. Based on a vertically constant wall temperature, similarity theory indicated that the width of the layer increased like $x^{1/4}$, where x is the vertical length measured from the bottom of the turbulent layer. Because of the divergence between the downward laminar flow and the upward turbulent flow, there was a thin region centered on the level at which the inner flow became turbulent where fluid flowed toward the ice. Further details are to be found in Josberger (1979) and Josberger and Martin (1980).

Unaware of Josberger's ongoing study, but motivated by an investigation by Neshbya (1977), Huppert and Turner (1978, 1980) conducted a series of experiments of melting iceblocks in a vertical salinity gradient. The rationale for the inclusion of a salinity gradient was that this mimics the situation in the ocean and that cooling a vertical salinity gradient from a side wall can lead to a convection pattern altogether different from the one that results if there is no salinity gradient. Huppert and Turner's experiments consisted of immersing iceblocks 0.2 m high in water stratified with salt at density gradients $\rho^{-1}d\rho/dz$ between 0.04 and 0.6 m^{-1} mainly at room temperature of around 20°C (though a few experiments were conducted at around 10°C).

A photograph of the flow pattern observed is shown in Fig. 2. Next to a thin inner layer of upward flowing meltwater, there was a thicker outer layer of downward flowing water. Because of the increase of salinity, and hence density, with depth the water in the cooled outer layer reached a level of neutral buoyancy where it flowed away from the ice. This water was relatively colder and fresher than the inward flowing water just beneath it, because it had been cooled both by diffusion and by the addition of meltwater and came originally from

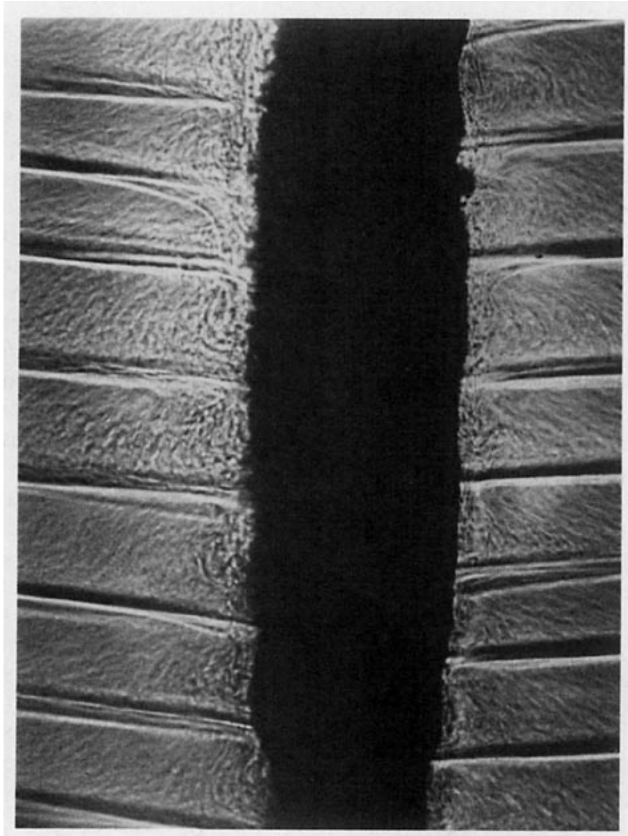

FIG. 2. A shadowgraph of ice melting in a salinity gradient. T_∞ = 19.3°C, $\rho^{-1}d\rho/dz$ = 0.11 m^{-1}, h = 18 mm (experiment 8).

a level of smaller salinity. Double-diffusive convection thus occurred between the two counterflowing layers, producing, even on the smallest laboratory scales, turbulence in the counterflowing layers and in the outer boundary layer. It is because the outer boundary layer was turbulent that it could entrain the meltwater in the inner boundary layer and eventually deposit it at a level not far from where it was formed. The heat transfer associated with the double-diffusive convection was responsible for the slight upward tilt of the interfaces apparent in Fig. 2. Aligned with the nearly horizontal layers, there were a series of horizontal ridges in the ice face due to the relatively small heat transfer to the ice at the level of the interfaces where there is little motion. The vertical scale of the layers in the experiments was related to the vertical density gradient by

$$h = 0.65[\rho(T_w,S_\infty) - \rho(T_\infty,S_\infty)]\left(\frac{d\rho}{dz}\right)^{-1} \quad (3)$$

$$\equiv 0.65\eta, \quad (4)$$

with $T_w = 0$. Arguing by analogy with their experiments on either heating or cooling a vertical salinity gradient from the side, Huppert and Turner suggested that (3) should be replaced by

$$h = f(\mathrm{G})\eta, \quad (5)$$

where the thermal Grashof number G, based on the vertical scale of the horizontal layers and thus incorporating the farfield density gradient, is given by

$$\mathrm{G} = g[\rho(T_w,S_\infty) - \rho(T_\infty,S_\infty)]^4\left(\frac{d\rho}{dz}\right)^{-3}(\rho_0\nu^2)^{-1}, \quad (6)$$

where ρ_0 is the mean density and the function f(G) falls steadily from 1.0 around G = 10^5 to approximately 0.65 around G = 10^6, whereafter it is constant. This form of f(G) could not be ascertained directly from Huppert and Turner's experiments with ice, which encompassed a range of G from 2.3 $\times 10^5$ to 1.2 $\times 10^9$, but was ascertained from their experiments on heating or cooling from the side, which covered a range of G = 1.2 $\times 10^4$ to 1.5 $\times 10^{10}$.

These two studies, with their very different flow fields, leave some doubt as to what to expect under oceanic conditions, at lower ambient temperatures, weaker salinity gradients and larger ice walls than were used in Huppert and Turner's experiments.

It might be argued (and it was by many) that because the change in density due to temperature is so much greater around 20°C than around 0°C, the downward buoyancy will decrease as the far-field temperature is decreased and experiments at low temperatures would display different flow patterns than those observed at 20°C. It was also suggested that a smaller temperature difference between the fresh meltwater and the ambient water would give greater prominence to dilution effects and to the salinity difference between the fresh inner layer and the far field. This might allow the inner layer to dominate and continue to flow upward, entraining ambient water—just as in Josberger's experiments. Additionally, the smaller vertical salinity gradient might weaken the outer boundary layer sufficiently to lead to a flow in closer similarity to that with no salinity gradient.

We thus conducted a series of experiments in which ice melted in water at (uniform) temperatures of between +1.9 and +4.9°C and with density gradients of the order of 10^{-2} m^{-1}, corresponding to a salinity gradient of the order of 10‰ m^{-1}. The experiments indicate that at these temperatures and density gradients the meltwater spreads out in a series of horizontal layers, just as at the higher temperatures and salinity gradients, with a scale well predicted by (3) with $T_w = T_{fp}$.

It is our opinion that laboratory experiments at salinity gradients comparable to those in the ocean would require considerable ingenuity. We hence argue in Section 3, when discussing the polar data, that the next step in the investigation of the deposition of meltwater from icebergs is to conduct a careful program of field observations.

2. The experiments

The experiments were carried out in Cold Room Number 1 at the Scott Polar Research Institute in Cambridge. Using the standard double-bucket filling technique of Oster and Yamamoto (1963) and Oster (1965), we filled a Perspex tank 0.50 m × 0.20 m × 0.40 m high to within 0.04 m of the top with water linearly stratified with salt. The water had been in the Cold Room for a sufficient time to be at approximately the same temperature as its environment. We then withdrew samples at 0.03 m intervals in order to subsequently evaluate the density gradient with the aid of a refractometer. Before each experiment we made an iceblock 0.19 m × 0.04 m × 0.50 m by freezing distilled water in a Flash Freezer. By placing a layer of Perspex on the bottom of the tray in which the ice was made and insulating the top of the tray so that freezing took place preferentially from the bottom, we obtained one very smooth face, in the vicinity of which there was little trapped air. The opposite face was often quite rough and a considerable amount of air remained trapped in the upper 0.02 m. An array of either eight or nine thermistors, with measuring heads of ~2 mm was frozen into the ice, with each thermistor at a slightly different distance from the bottom face, but all within 0.01 m of that face. We believe that the absolute error of the thermistor readings was +0.1°C, although the comparative error during each experiment would be rather less. We allowed the ice to come to a uniform temperature of approximately 0°C before turning it through 90° and inserting it into the tank with its two largest faces vertical. The ice touched the bottom and extended ~0.1 m above the surface. We observed the subsequent motions in the water by using the shadowgraph technique and by the occasional dropping of dye from the surface. Table 1 summarizes the relevant parameters of the experiments and some of the quantitative results. It should be noted that the penultimate experiment was carried out at a warmer temperature than the others and that the last experiment used a slightly different procedure and will be discussed separately at the end of this section.

TABLE 1. The parameters and some of the results of the experiments. Note that experiment 9 had a region of uniform salinity beneath the stratified region; the tabulated quantities pertain to the stratified region. The thermistor array was not used in experiments 1 and 2. In experiments 6 and 7 there was a vertical variation of about 3°C in T_w.

Experiment No.	T_∞ (°C)	$\rho^{-1}d\rho/dz$ (m^{-1})	S_∞ (‰)	h (mm)	T_{fp} (°C)	T_w (°C)	h/η_{fp}	h/η_w	G
1	4.9	0.066	47.9	9	−2.7		0.85		4.6×10^5
2	2.4	0.040	64.6	10	−3.7		0.57		2.1×10^6
3	2.4	0.028	53.2	13	−3.0	−2.4	0.75	0.82	1.4×10^6
4	3.3	0.012	38.6	15	−2.2	−1.7	0.54	0.55	4.0×10^6
5	2.4	0.0089	37.5	17	−2.1	−1.6	0.63	0.69	2.6×10^6
6	4.4	0.20	90.5	7	−5.2		0.78		7.0×10^5
7	1.8	0.15	88.8	7	−5.1		0.85		3.8×10^5
8	19.3	0.11	41.1	18	−2.3	−0.3	0.54	0.55	7.7×10^7
9	3.2	0.012	35.5	15	−2.0	−1.0	0.64	0.72	2.0×10^6

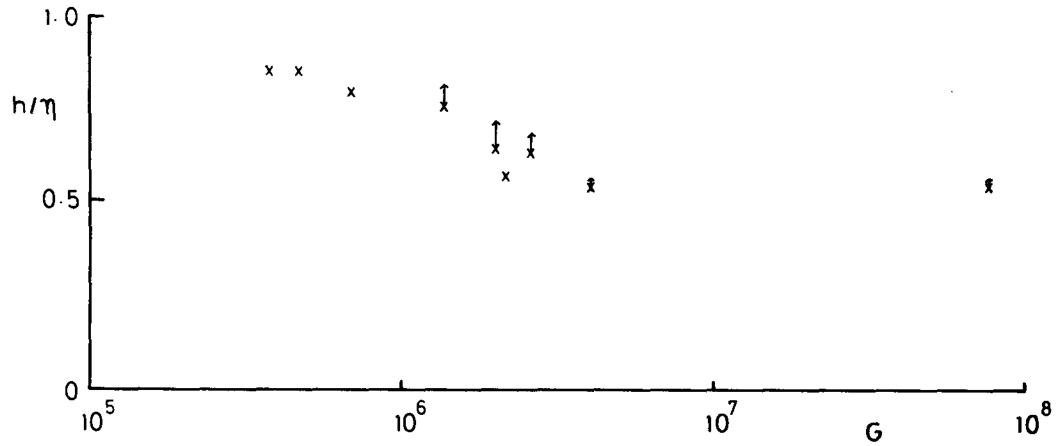

FIG. 3. The plot of h/η as a function of G. The crosses (×) represent h/η_{fp} and the end of the arrow heads h/η_w.

Upon insertion of the ice, a layer of upward flowing water of a few millimeters thickness appeared within a few seconds. The lowest 0.1 m or so of the layer was laminar, the upper 0.3 m was turbulent. After some 5 or 10 min the intensity of the upflow greatly diminished and horizontal layers began to be observable in the shadowgraph. These layers continued to develop horizontally and persisted for the duration of each experiment of up to 4 h. During each experiment a series of horizontal ridges appeared in the ice face, with a vertical spacing equal to that of the layers in the surrounding water. The motion in the layers was exactly as seen in the Huppert and Turner experiments at higher temperatures and salinity gradients, and described in the Introduction. Table 1 presents h, the vertical scale of the layers, h/η_{fp} and h/η_w, where the subscripts fp and w indicate that η [defined by Eq. (4)] has been evaluated by substituting for T_w the value of T_{fp} and the measured wall temperature, respectively, and S_∞ is the mean farfield salinity. We note that η_{fp} is only slightly larger than η_w because of the comparatively small change of density with temperature around T_{fp} at fixed salinity. The two quantities h/η_{fp} and h/η_w for the experiments are plotted as a function of the Grashof number [defined by Eq. (6)] in Fig. 3. We see that the data are consistent with the relationship obtained previously by Huppert and Turner (1979). We would have liked to conduct experiments with much weaker salinity gradients, which would have led to higher Grashof numbers but this did not seem possible.

From the thermistor array in the ice we obtained a record of temperature as a function of time at different heights. A typical result taken from experiment 1 is presented in Fig. 4. On inserting the ice at 0°C into salty water at a higher temperature, the temperature in the ice rapidly decreased to a fairly uniform value, independent of depth.[1] Each thermistor continued to record this value, taken as T_w, until it began to protrude beyond the ice; thereafter the temperature rose. Qualitatively, the same sort of record was obtained for experiment 8 with $T_\infty = 19.3$°C. Quantitatively, the wall temperature in this case was -0.3°C. This gives support to Huppert and Turner's (1979) use of 0°C rather than T_{fp} as the wall temperature in interpreting their results on ice melting in warm water.[2] Precise measurements of temperature as a function of distance from the ice face and of position with respect to the horizontal layers would require much smaller thermistor heads than we had available.

The measured wall temperatures were considerably closer to T_{fp} than that predicted by (2), the relationship obtained by Josberger for the wall temperature adjacent to the turbulent upward flowing region in the unstratified situation. We explain this by noting that stratification inhibits vertical motion, and thus decreases the velocity scale and melt rate (the latter by up to an order of magnitude, we estimate). The wall temperature is lowered toward the farfield freezing point as a result of the smaller melt rate.

[1] The thermal diffusivity of ice is 1.2 mm^2 s^{-1}; thus the time scale to reach thermal equilibrium in a 40 mm thick piece of ice is of the order of 5 min. The thermistors were initially at most 5 mm from the ice face, which implies that in less than one minute they reacted to a temperature change in the fluid adjacent to the wall. Thus, because the melting required many hours, we can safely equate the temperatures measured by the thermistors while still embedded in the ice to T_w, the temperature at the ice face.

[2] Note, however, that at oceanic values of S_∞, for $T_\infty \geqslant 5$°C, there is little *quantitative* difference between h evaluated from (3) with $T_w = T_{fp}$ and that with $T_w = 0$°C. This is because of the rapid increase of density with temperature away from 0°C.

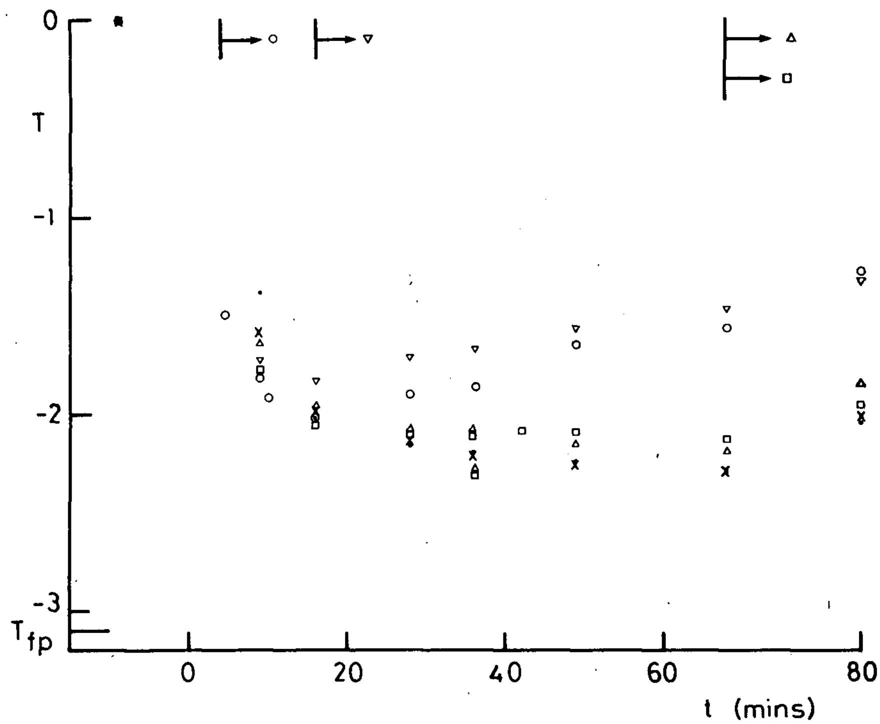

FIG. 4. The temperature as a function of time for experiment 1. The height of the termistors above the bottom are as follows: (●) 90 mm, (×) 150 mm, (△) 210 mm, (□) 240 mm, (○) 270 mm and (▽) 330 mm. Each vertical line and symbol at the top of the figure indicates the time at which the ice melted beyond the thermistor represented by that symbol.

Aside from the horizontal ridges, on the rougher face there frequently was a series of straight vertical grooves extending from top to bottom across the entire face. The grooves were about 3 mm wide and of the order of 0.5 mm deep near the bottom of the ice, gradually increasing to around twice this depth at the top. We attribute these grooves to the effect of the rising bubbles trapped in the ice

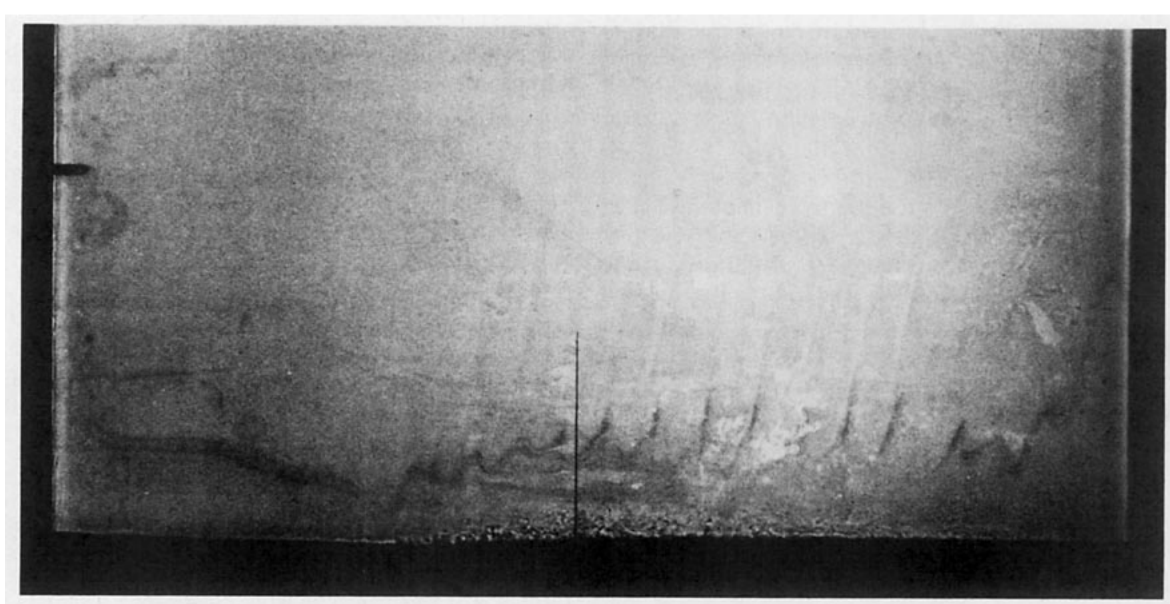

FIG. 5. A shadowgraph view of experiment 9 after 35 min. The horizontal line marks the top of the region of uniform salinity. The dye trace indicates the almost horizontal layering in the upper stratified region. In the lower region, the dye flows toward the ice at the level where the inner layer becomes turbulent and away from the ice at the bottom.

during the freezing process. It is interesting to note that while the bubbles affected the melt pattern in the ice, they had no visible effect on the motion away from the ice, as also found by Josberger (1980).

Experiment 9 was rather different. To investigate whether a turbulent upward flowing layer in a lower unstratified region would upset the formation or size of the horizontal layers in an upper stratified region, we filled the lower 0.175 m of the tank with water at a uniform density of 1.0246 gm cm^{-3} and then filled the tank to 0.354 m with a linear stratification. There was no density discontinuity at the interface. We found that the formation of layers in the stratified region proceeded exactly as previously observed, with a layer scale given by (3), with only the following differences. The turbulent layer continued all the way up to the ice face in the stratified region and the horizontal layers started further out from the ice than in the earlier experiments. Furthermore, the ice showed no evidence of ridging. A photograph of the flow pattern made visible by both shadowgraph and dye appears in Fig. 5. The inward flow in the unstratified region and the layering in the stratified region are clearly evident. It would be interesting and instructive to conduct a similar series of experiments on a larger scale; an increase in depth of even a factor of 3 would be useful. Unfortunately, a deeper tank and the facilities for making larger iceblocks were not available to us.

We conclude this section with a discussion of two further aspects of our experiments. First, in the two experiments with large salinity gradient, numbers 6 and 7, the thermistors recorded temperatures that decreased with depth both in the ice and in the water just next to the ice. In each case the temperature difference from top to bottom was around 3°C. The freezing point depression also decreased with depth, but at a slightly larger rate. Thus, the insertion of ice into salt-stratified water, with both ice and water originally at a uniform temperature, leads to a vertical temperature gradient in the ice and in the water. The second aspect is the formation of an overhang and undercut structure in the ice at the water surface as shown in Fig. 6. We think the structure occurred because the air at about 1°C warmed the surface salt water. The undercutting results from the flow of the warmed surface water toward the ice due to the motion in the uppermost layer. The fresh meltwater from that portion of the ice above the surface drained onto the salt water and froze to form the overhang. This freezing occurred because the subsurface melting had cooled the surface salt water, and the ice a small distance above the water, to a temperature below the fresh water freezing point.

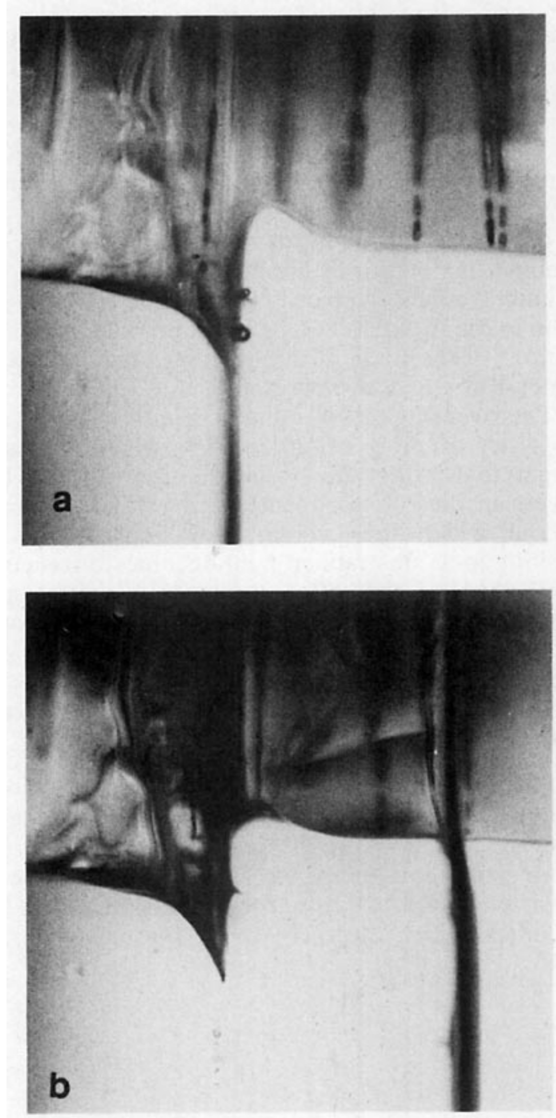

FIG. 6. (a) The overhang and undercut region at the intersection of the water surface and the ice wall. To obtain (b) the water level was lowered 1.5 cm.

3. Discussion

To apply our results to polar oceans it is necessary to assume that it is valid to extrapolate them to smaller salinity gradients than it is possible to investigate in the laboratory. Data from the Antarctic indicate ambient temperatures of around 0°C and salinity gradients in the summer varying between 10^{-3} ‰ m^{-1} in the upper 100 m of the Weddell Sea and 10^{-5} ‰ m^{-1} in a relatively unstratified region down to 400 m (Foster and Carmack, 1976). Considering a nominal 1°C temperature difference between the ice and the ocean and using Eq. (3), we calculate a corresponding layer scale of order 10 m in the upper 100 m and 10^3 m below that. The latter value indicates that the influence of the salinity gradient is inconsequential at depth, and convection up the side of the ice occurs, as in Josberger's unstratified experiments. From experiment 9, which mimics the stratification both above and below 100 m, we expect that the convection penetrates through to the upper region and that horizontal layers form there.

In the upper 30 m of the Arctic Ocean the ambient temperatures are also around 0°C and the salinity gradient is typically 10^{-2} ‰ m^{-1}, rather larger than in the Antarctic (Hellerman, private communication). This gradient implies that layer scales of order 1 m are to be expected.

The overall Grashof number [Eq. (1)] for layers of order 10 m is of order 10^{13}, which is much larger than either the value of order 10^5 for the layers in these experiments or the value of order 10^8 at which the laminar flow induced by ice melting in an unstratified fluid becomes turbulent.

Because of the different parameter range and scale in the ocean, unobtainable in the laboratory, we suggest that the next step to validate our ideas requires a field program of observations to examine the temperature, salinity and velocity structure in the vicinity of icebergs.

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